APPENDIX B

SAMPLE CALCULATIONS USING RUN G22L16

Calculation of Pressure Gradient

The pressure gradient was calculated from data obtained using the following conditions:

$$D_c = 15.2 \text{ cm},$$

$$U_G = 3.86 \text{ cm/sec},$$

$$\gamma_o = 4.13 \text{ mmho}.$$

For U_L = 0, the pressure drop readings obtained were 34.5- , 34.4- , 34.4- , 34.4- , 34.3- , 34.3- , 34.4- , 34.5- , 34.5- , 34.7- , and 34.6-cm water for m_1 through m_{11}, respectively.

The pressure drop readings obtained for U_L = 5.14 cm/sec were 58.0- , 53.4- , 48.8- , 44.2- , 39.3- , 34.8- , 33.3- , 33.9- , 34.7- , 35.9- , and 36.6-cm water for m_1 through m_{11}, respectively.

The conductivity readings obtained for U_L = 5.14 cm/sec were 1.67, 1.78, 1.73, 1.68, 1.68, 2.79, 3.59, 3.61, 3.62, and 3.57 mmho at column heights of 5, 13, 21, 29, 37, 45, 53, 61, 69, and 77 cm, respectively.

Calculation of the pressure drops as shown in the previous section (Appendix A) gave 4.5- , 9.1- , 13.7- , 18.5- , 23.0- , 24.6- , 24.1- , 23.3- , 22.3- , and 21.5-cm water for Δh_2 through Δh_{11}, respectively.

The pressure taps 1-11 in the 15.2-cm-ID column are located along the wall at heights of 1, 9, 17, 25, 33, 41, 49, 57, 65, 73, and 80 cm, respectively, above the bottom of the bed. Fitting straight lines to the pressure drops in and above the bed results in a calculated bed height of 44.75 cm.

The total pressure drop at any point in the column is the pressure drop due to flow plus the static head. Thus the total pressure drop is calculated from the following:

$$\Delta P_i = \rho_L \, g \, (\Delta h_i + T_i - T_1) \quad , \qquad (48)$$

where

ΔP_i = total pressure drop between pressure taps i and 1,

T_i = height above column bottom for pressure tap i.

Calculation of the total pressure drops then gives the following:

ΔP_2 = (0.995) (981) (4.5 + 9 - 1) = 12201.2 dyn/cm²,

ΔP_3 = (0.995) (981) (9.1 + 17 - 1) = 24500.0 dyn/cm²,

ΔP_4 = (0.995) (981) (13.7 + 25 - 1) = 36798.8 dyn/cm²,

ΔP_5 = (0.995) (981) (18.5 + 33 - 1) = 49292.8 dyn/cm²,

ΔP_6 = (0.995) (981) (23.0 + 41 - 1) = 61494.0 dyn/cm²,

ΔP_7 = (0.995) (981) (24.6 + 49 - 1) = 70864.5 dyn/cm²,

ΔP_8 = (0.995) (981) (24.1 + 57 - 1) = 78185.2 dyn/cm²,

ΔP_9 = (0.995) (981) (23.3 + 65 - 1) = 85213.1 dyn/cm²,

ΔP_{10} = (0.995) (981) (22.3 + 73 - 1) = 92045.8 dyn/cm²,

ΔP_{11} = (0.995) (981) (21.5 + 80 - 1) = 98097.5 dyn/cm².

The calculated bed height was found to be 44.75 cm. Thus, pressure taps 1-6 were located in the bed, while taps 7-11 were situated above the bed. Performing a least-squares linear fit to the total pressure drops in the bed as a function of height results in the following equation:

$$\Delta P_{\text{in bed}} = -1698.4 + 1542.2\ h \quad . \tag{49}$$

Pressure Gradient Method for Calculation of Overall Holdups

The solid holdup can be determined by using Eq. (3):

$$\varepsilon_S = \rho_S AH \quad . \tag{3}$$

Substitution of numerical values yields:

$$\varepsilon_S = (9556.3)/(2.243)(182.4)(44.75) = 0.522.$$

Equation (2) is:

$$\Delta P = gH(\varepsilon_L\rho_L + \varepsilon_G\rho_G + \varepsilon_S\rho_S) \quad . \tag{2}$$

Therefore,

$$d(\Delta P)/d(H) = g(\varepsilon_L\rho_L + \varepsilon_G\rho_G + \varepsilon_S\rho_S) \quad , \tag{50}$$

assuming that the holdups are constant over the entire bed and that $d(\Delta P)/d(h)$ is the slope of the ΔP-versus-h line. Thus, substitution into the above yields:

$$1542.2 = 981[0.995\,\varepsilon_L + 0.0013\,\varepsilon_G + 2.243(0.522)],$$
$$1.572 = 0.995\varepsilon_L + 0.0013\,\varepsilon_G + 1.171,$$
$$0.401 = 0.995\varepsilon_L + 0.0013\,\varepsilon_G .$$

Substituting into Eq. (1) gives:

$$1 = \varepsilon_L + \varepsilon_G + \varepsilon_S,$$

$$1 = \varepsilon_L + \varepsilon_G + 0.522.$$

Therefore,

$$\varepsilon_L + \varepsilon_G = 0.478.$$

Solving these two equations simultaneously then yields the remaining two holdups:

$$0.401 = 0.995\,\varepsilon_L + 0.0013\,\varepsilon_G,$$

$$0.478 = \varepsilon_L + \varepsilon_G.$$

Therefore,

$$\varepsilon_L = 0.403,$$

$$\varepsilon_G = 0.075,$$

$$\varepsilon_S = 0.522.$$

<u>Conductivity Method for Determination of Overall Holdups</u>

Equation (7), in which the conductivity is measured at the middle of the bed, can also be used to calculate the overall holdups;

$$\varepsilon_L = \gamma/\gamma_o \quad . \tag{7}$$

The bed height is 44.75 cm, and the conductivity at the middle of the bed is 1.73 mmhos. Thus,

$$\varepsilon_L = 1.73/4.13 = 0.419.$$

Substituting into Eq. (2) yields:

$$1542.2 = 981[0.995(0.419) + 0.0013\ \varepsilon_G + 2.243\ \varepsilon_S],$$

$$1.572 = 0.417 + 0.0013\ \varepsilon_G + 2.243\ \varepsilon_S,$$

$$1.155 = 0.0013\ \varepsilon_G + 2.243\ \varepsilon_S.$$

Substituting into Eq. (1) gives:

$$1 = 0.419 + \varepsilon_G + \varepsilon_S,$$

or

$$\varepsilon_G + \varepsilon_S = 0.581.$$

Solving these two equations simultaneously then yields the remaining two holdups:

$$1.155 = 0.0013\ \varepsilon_G + 2.243\ \varepsilon_S,$$

$$0.581 = \varepsilon_G + \varepsilon_S.$$

Therefore,

$$\varepsilon_L = 0.419,$$

$$\varepsilon_G = 0.066,$$

$$\varepsilon_S = 0.515.$$

<u>Calculation of Incremental Holdups</u>

Equation (7) can be used to calculate the liquid holdup as a function of height in the column:

$$\varepsilon_L = \gamma/\gamma_o \quad . \tag{7}$$

At $h = 5$, $\varepsilon_L = 1.67/4.13 = 0.404$,

At $h = 13$, $\varepsilon_L = 1.78/4.13 = 0.431$,

At $h = 21$, $\varepsilon_L = 1.73/4.13 = 0.419$, At $h = 29$, $\varepsilon_L = 1.68/4.13 = 0.407$,

At $h = 37$, $\varepsilon_L = 1.68/4.13 = 0.407$,

At $h = 45$, $\varepsilon_L = 2.79/4.13 = 0.676$,

At $h = 53$, $\varepsilon_L = 3.59/4.13 = 0.869$,

At $h = 61$, $\varepsilon_L = 3.61/4.13 = 0.874$,

At $h = 69$, $\varepsilon_L = 3.62/4.13 = 0.887$,

At $h = 77$, $\varepsilon_L = 3.57/4.13 = 0.864$.

Substitution into Eq. (10) for just the column increment around the column height of 5 cm results in:

$$dP/dh = g(\varepsilon_L\rho_L + \varepsilon_G\rho_G + \varepsilon_S\rho_S) \quad , \qquad (10)$$

$$(\Delta P_2 - \Delta P_1)/(T_2 - T_1) = g(\varepsilon_L\rho_L + \varepsilon_G\rho_G + \varepsilon_S\rho_S) \quad , \qquad (51)$$

or

$$(12201.2-0)/(9-1) = 981[0.995(0.404) + 0.0013\varepsilon_G + 2.243\varepsilon_S],$$

$$1.153 = 0.0013\,\varepsilon_G + 2.243\varepsilon_S \ .$$

Substitution into Eq. (1) yields:

$$1 = 0.404 + \varepsilon_G + \varepsilon_S ,$$

$$0.596 = \varepsilon_G + \varepsilon_S .$$

Simultaneous solution of these two equations then gives:

$$1.153 = 0.0013\,\varepsilon_G + 2.243\varepsilon_S,$$

$$0.596 = \varepsilon_G + \varepsilon_S .$$

Therefore,

$$\varepsilon_G = 0.082,$$

$$\varepsilon_S = 0.514.$$

Substitution into Eq. (10) for the column increment around the column height of 13 cm yields:

$$(\Delta P_3 - \Delta P_2)/(T_3 - T_2) = g(\varepsilon_L \rho_L + \varepsilon_G \rho_G + \varepsilon_S \rho_S) \quad , \qquad (52)$$

or

$$(24500-12201.2)/(17-9) = 981[0.995(0.431)+0.0013\varepsilon_G - 2.243\,\varepsilon_S],$$

$$1.138 = 0.0013\,\varepsilon_G + 2.243\,\varepsilon_S.$$

Substitution into Equation (1) gives:

$$1 = 0.431 + \varepsilon_G + \varepsilon_S,$$

$$0.569 = \varepsilon_G + \varepsilon_S.$$

Therefore,

$$\varepsilon_G = 0.062,$$

$$\varepsilon_S = 0.507.$$

In this manner, all of the holdups can be obtained as a function of height in the column, as shown in Table 9.

<u>Fit of Local Incremental Holdups</u>

The local holdups for Run G22L15 for U_L = 5.14 cm/sec are plotted as a function of column position in Fig. 35 (see page 89). Each holdup curve was fitted using the error function as described previously. The constant portion of the holdups was calculated by determining the best horizontal fit to the gas and solid holdup curves, starting at the bottom (or top) of the column and progressing toward the middle of the column, one point at a time. Points were

Table 9. Calculation of holdups as a function of position within the column.

Column height (cm)	Liquid holdup	Gas holdup	Solid holdup
5	0.404	0.082	0.514
13	0.431	0.062	0.507
21	0.419	0.068	0.513
29	0.407	0.064	0.529
37	0.407	0.081	0.513
45	0.676	0.092	0.233
53	0.869	0.131	0.030
61	0.874	0.126	0.011
69	0.877	0.123	-0.001
77	0.864	0.136	0.009

continually added until one was found that gave a poor horizontal fit. Consequently, the gas holdup in the bed was determined using the lowest five points:

$$\varepsilon_G''' = (0.082 + 0.062 + 0.068 + 0.064 + 0.081)/5 = 0.072.$$

Similarly, the gas holdup above the bed was determined using the uppermost four points:

$$\varepsilon_G'' = (0.136 + 0.123 + 0.126 + 0.131)/4 = 0.129.$$

The solid holdup in the bed, on the other hand, used only the lowest three points since addition of the fourth point gave a poor horizontal fit:

$$\varepsilon_S''' = (0.514 + 0.507 + 0.513)/3 = 0.511.$$

Once the constant portions of the holdup curves are determined, an average is calculated. For the gas holdup curve, this average was:

$$\varepsilon_{G,avg} = (0.072 + 0.129)/2 = 0.1005.$$

The error function is then fitted to two points on either side of this average, using the point of inflection, I_G, and a standard deviation, σ_G, as follows:

$$\begin{aligned} \mathrm{erf}(Y_A) &= (\varepsilon_{G,A} - \varepsilon_{G,avg})/(\varepsilon_{G,avg} - \varepsilon_G''') \quad , \qquad (53) \\ &= (0.131-0.1005)/(0.1005-0.072) = 1.0702; \end{aligned}$$

$$\begin{aligned} \mathrm{erf}(Y_B) &= (\varepsilon_{G,B} - \varepsilon_{G,avg})/(\varepsilon_{G,avg} - \varepsilon_G''') \quad , \qquad (54) \\ &= (0.092-0.1005)/(0.1005-0.072) = -0.2982; \end{aligned}$$

where

$\varepsilon_{G,A}$ = the local gas holdup at the first point above $\varepsilon_{G,avg}$,

$\varepsilon_{G,B}$ = the local gas holdup at the first point below $\varepsilon_{G,avg}$.

The terms Y_A and Y_B are then found by taking the inverse error function of 1.0702 and -0.2982, respectively. Since the error function cannot be greater than 1 or smaller than -1, any such values are set equal to 0.9999 or -0.9999. Then,

$$Y_A = \mathrm{erf}^{-1}\,[\mathrm{erf}(Y_A)] \quad , \tag{55}$$
$$= \mathrm{erf}^{-1}(0.9999) = 2.765;$$

$$Y_B = \mathrm{erf}^{-1}\,[\mathrm{erf}(Y_B)] \quad , \tag{56}$$
$$= \mathrm{erf}^{-1}(-0.2982) = -0.265.$$

The standard deviation around $\varepsilon_{G,avg}$ is given by:

$$\sigma_G = (h_A - I_G)/Y_A = (h_B - I_G)/Y_B \quad , \tag{57}$$

where

h_A = position of first point where $\varepsilon_G > \varepsilon_{G,avg}$,

h_B = position of first point where $\varepsilon_G < \varepsilon_{G,avg}$.

Solving the two equations simultaneously gives I_G :

$$I_G = (Y_A\, h_B - Y_B\, h_A)/(Y_A - Y_B) \quad , \tag{58}$$
$$=[(2.765)(45)-(-0.265)(53)]/[2.765-(-0.265)]=45.7 \text{ cm.}$$

Then, the standard deviation can be found by using one of the above equations:

$$\sigma_G = (53-45.7)/(2.765) = 2.64 \text{ cm.}$$

The gas holdup curve is then fitted using Eq. (32):

$$\varepsilon_G = [(P_G - 1)/-2]\varepsilon_G''' + [(P_G + 1)/2]\varepsilon_G'' \quad , \tag{32}$$

where

$$P_G = \text{erf } [(h - I_G)/\sigma_G] \quad . \tag{33}$$

For a column position of 10 cm above the bottom of the bed:

$$P_G = \text{erf}[(10-45.7)/2.64)] = \text{erf}(-13.523)=-1,$$

$$\varepsilon_G = [(-1-1)/-2]0.072 + [(-1+1)/2]0.129 = 0.072.$$

For a column position of 47 cm above the bottom of the bed:

$$P_G = \text{erf}[(47-45.7)/2.64] = \text{erf}(0.492) = 0.516,$$

$$\varepsilon_G = [(0.516-1)/(-2)]0.072 + [(0.516 + 1)/2]0.129,$$

$$\varepsilon_G = 0.017 + 0.098 = 0.115.$$

For a column position of 70 cm above the bottom of the bed:

$$P_G = \text{erf}[(70-45.7)/2.64] = \text{erf}(9.205) = 1,$$

$$\varepsilon_G = [(1-1)/(-2)]0.072 + [(1+1)/2]0.129 = 0.129.$$

The solid holdup curve is fitted in the same manner, remembering that $\varepsilon_S'' = 0$. The liquid holdup curve is then fitted as the residual of Eq. (1).