

RECEIVED

SEP 11 1998

OSTI

**Measurement Techniques for Local and Global Fluid
Dynamic Quantities in Two and Three Phase Systems**

**Topical Report
January 1998**

**By:
S. K. Kumar; M. P. Duduković
B. A. Toseland**

Work Performed Under Contract No.: DE-FC22-95PC95051

For
U.S. Department of Energy
Office of Fossil Energy
Federal Energy Technology Center
P.O. Box 880
Morgantown, West Virginia 26507-0880

MASTER

JAT

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED
By
Air Products and Chemicals, Inc.
Allentown, Pennsylvania

Disclaimer

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible electronic image products. Images are produced from the best available original document.

Contents

Abstract	iii
Executive Summary	iv
1 Introduction	1
2 Gas Holdup and Solids Concentration Measurement	1
2.1 Global Techniques	2
2.2 Line Average Measurements based on Radiation Techniques	6
2.3 Probes for Local Gas Holdup and Solids Concentration Measurement	16
2.3.1 Impedance Probes	16
2.3.2 Optical Probes	19
2.3.3 Other Probes	20
3 Measurement of Bubble Sizes and Velocity	22
4 Measurement of Liquid and Solid Velocities	29
5 Final Recommendations and Remarks	37
6 Nomenclature	39
7 References	41

Abstract

Available measurement techniques for evaluation of global and local phase holdups, instantaneous and average phase velocities and for the determination of bubble sizes in gas-liquid and gas-liquid-solid systems are reviewed. Advantages and disadvantages of various techniques are discussed. Particular emphasis is placed on identifying methods that can be employed on large scale, thick wall, high pressure and high temperature reactors used in the manufacture of fuels and chemicals from synthesis gas and its derivatives.

Executive Summary

This report presents a critical review of the methods available for assessing the fluid dynamic parameters in large industrial two and three phase bubble column and slurry bubble column reactors operated at high pressure and temperature. The physical principles behind various methods are explained, and the basic design of the instrumentation needed to implement each measurement principle is discussed. Fluid dynamic properties of interest are : gas, liquid and solids holdup and their axial and radial distribution as well as the velocity distribution of the two (bubble column) or three phases (slurry bubble column). This information on operating pilot plant and plant reactors is essential to verify the computational fluid dynamic codes as well as scale-up rules used in reactor design. Without such information extensive and costly scale-up to large reactors that exploit syngas chemistries, and other reactors in production of fuels and chemicals, cannot be avoided. The fluid dynamic parameters mentioned above must be measured under operating conditions in large units and the data used for evaluating existing models.

The findings of this study can be summarized as follows. While many promising techniques based on conductance, impedance, ultrasound, light transmittance and other principles are emerging, none has yet reached the status of being capable of providing reliable measurements in high temperature, high pressure, large diameter columns operated in the churn turbulent regime with high gas and solids holdup. Hence, further development of these techniques in the laboratory should be encouraged.

The following methods seem most suitable at the moment for providing the fluid dynamic parameters in industrial columns such as the Laporte AFDU reactor.

For global gas holdup measurements, determination of the dynamic height of the slurry in an operating reactor (by a gamma ray or other means) and of the settled slurry in the absence of gas provides the least ambiguous value of overall gas holdup. During shut down the dynamic gas disengagement technique is capable of providing global information on the existence of unimodal or multimodal bubble size distribution. Overall solids holdup can be determined from the volume of the solids loaded and the height (volume) of the slurry at operating conditions.

Best estimates of gas holdup during operation, and its distribution along the column height, can be obtained via sectional pressure drop measurements. This estimate requires the knowledge of the solids concentration distribution which is not always available.

Density profiles along the column can be obtained by gamma ray densitometry (a needed correction for obtaining proper cross sectional values from the usually measured chordal averages is discussed in this report). Cross-sectional density distribution can be readily obtained

by gamma ray tomography. Obtaining gas holdup profiles along the column height, or in a cross section, again requires information (or assumptions) regarding solids concentration distribution. Solids concentration profiles can be obtained noninvasively by dual beam gamma ray tomography using sources with photon emissions at two different energy levels. This may require excessive scanning times. Invasive isokinetic sampling or point conductance probes represent the unappealing alternative in hot and high pressure systems for the measurement of local velocity and/or phase concentration. Ultrasound probes deserve further attention and can be suitably designed to provide local measurements of phase concentrations.

While sophisticated techniques such as Particle Image Velocimetry (PIV) and Computer Aided Radioactive Particle Tracking (CARPT) can be used in the laboratory to map the velocity fields, at present their use in industrial reactors is not possible. To determine liquid velocities, pitot tubes seem to be still favored, while gas (bubble) velocities can be assessed by multipoint optical probes. How well these can work in systems with high solids loading remains to be demonstrated. Only CARPT is capable of determining solids velocities at high solids holdup, and this, unfortunately, cannot be implemented for large reactors.

Specific recommendations for the Laporte reactor can be summarized as follows :

1. Continue to use gamma densitometry and implement gamma ray tomography for non-invasively obtaining density distributions along and across the column at all operating conditions.
2. Develop if possible dual beam densitometry by adding the Am-241 or Co-60 source to the current Cs-137 source. This would enable one to resolve the gas, liquid and solids holdup profiles in the column at all operating conditions.
3. Install additional pressure taps for the measurement of sectional pressure drop along the column, and also monitor pressure fluctuations and compute cross-correlations between various points.
4. Prior to each shutdown execute the dynamic gas disengagement method and evaluate the bubble size distribution.
5. Consider installing invasive probes for the estimation of phase velocities. Pitot tube, five point optical probes and heat transfer probes are the current likely candidates.

1 Introduction

Design and scale-up of bubble columns, slurry bubble columns, three phase fluidized beds and other multiphase systems are still predominantly based on empirical correlations validated over a limited range of operating conditions and physical properties. Application of more fundamental fluid dynamic models awaits their experimental verification. Hence, measurement of fluid dynamic quantities such as phase velocities, phase holdups, bubble size etc. are of great interest both for extending the range of validity of current correlations and for verification of fundamental hydrodynamic models. Measurement is also important in interpreting, understanding and predicting reactor performance. Advances in instrumentation and signal processing techniques have led to a rapid proliferation of the available experimental methods for measurements in multiphase flows. There are a number of good reviews of these methods in the literature including those by Banerjee and Lahey (1981), Delhaye (1986), Jones and Delhaye (1976), Snoek (1990) and Lubbert, (1991) to mention a few.

An attempt is made here to present a comprehensive review of the experimental methods available for multiphase systems. The emphasis is on techniques that can be utilized under conditions of interest in practice such as high pressure and temperature, large solids holdup etc. The measurement methods for the determination of the following quantities are considered :

- Void fraction and solids concentration;
- Bubble size distribution and rise velocities;
- Liquid and solid phase velocities.

2 Gas Holdup and Solids Concentration Measurement

Gas holdup and solids concentration measurement methods can be broadly classified into two categories : (1) those providing an overall or global measurement and (2) those that provide a local or point measurement. The global measuring techniques yield information on the line, area or volume averaged gas or solids holdup. The volume averaged or overall fractional holdup of a phase is defined as the fraction of the volume of the multiphase dispersion that is occupied by that phase. In general, the measurement of the overall holdup is relatively simple. It provides information regarding what fraction of the system volume is occupied by the phase of interest. Measurement of the phase fraction at a point is in itself not of much use unless the distribution of such point measurements is obtained in space. If such

point measurements are obtained using some kind of invasive probe (as is done most often), then obtaining the distribution of the local phase volume fraction involves a tedious data collection process. This, in addition to their intrusive nature constitutes a serious drawback in using probes to obtain the local phase holdup in two or three phase flow systems.

2.1 Global Techniques

The simplest method of measuring the overall fractional gas holdup in two phase systems is rather straight forward. The knowledge of the volume occupied by the phase of interest as well as the volume of the expanded multiphase dispersion suffices for the needed calculation. For a system with uniform cross-section the process involves the measurement of the height of the dispersion H_g at operating conditions and that of the static column (with no gas flow), H_0 , and calculating the gas holdup from :

$$\epsilon = (H_g - H_0) / H_g \quad (1)$$

Frequently an accurate measurement of the height of the dispersion is not possible either due to the fluctuation of the level of the free surface due to bubble disengagement (especially at high gas flow rates) or due to the formation of foam at the free surface of the dispersion. Generally, an average of several observed readings is adopted to minimize the error in the measurement. Although, most often the height of the dispersion is obtained directly, with the aid of a length scale, other methods have also been used. A novel method involving the use of a hot film anemometer has been used by Deckwer et. al. (1980) for determination of the free surface of the dispersion in a slurry bubble column. The hot wire was first calibrated in the gas and the slurry phase. With the top of the column as reference, the hot wire is moved down towards the suspension and, when it touches the slurry phase, the heat removal and, hence, the electric current, increases sharply thereby enabling an accurate measurement of the local height of the dispersion. Clark (1987) has used a neutron based technique for detecting the free surface of the slurry in a high pressure bubble column. The method is based on the ability of the liquid phase to scatter more neutrons than the gas. The gas liquid interface is detected by moving the assembly of the neutron source and detector down the reactor until a decrease in the neutron flux is detected. The precision of interface detection is reported to be $\pm 5\text{mm}$. The method can also be implemented using a γ ray source instead of the neutron beam. All these methods, however, still contain an error associated with the fluctuation of the free surface. In addition, the measurement made of the free surface is local and instantaneous. Consequently, a true average can be obtained by making the measurement at different points and at different instants of time.

For non-transparent columns, as in most industrial situations, the position of a float can be used for detecting the free surface. The position of the float, as it moves with the free surface of the dispersion, is transmitted either through a system of linkages or a string and pulley arrangement to a pointer moving along a length scale. This method has the advantage in that the float dampens out some of the fluctuations of the free surface of the dispersion.

Another method that has been adopted for obtaining the overall fractional phase holdup in two phase systems is to infer it from the measurements of the pressure drop along the height of the column. The theoretical background for the technique has been presented by Hills (1976), as well as by Merchuk and Stein (1981). In gas-liquid systems, if the liquid flow rate is small then, to a good approximation, the pressure drop can be wholly attributed to the hydrostatic head as :

$$\frac{dP}{dh} = -\rho_l g (1 - \epsilon) \quad (2)$$

where h is the axial coordinate pointing upwards and ρ_l is the liquid (two phase system) or slurry (three phase system) density. The actual measurement can either be of the static pressure, or of the differential pressure using manometers or some kind of a pressure transducer. If dz represents the height difference observed in a differential manometer, the following expression holds :

$$\rho_l g \frac{dh}{dz} = \frac{dP}{dz} + \rho_l g \quad (3)$$

From these expressions it follows that

$$\epsilon = \frac{dz}{dh}$$

If, however, the liquid flow rate is not negligible, one has to account for the wall shear stress and for the effects of an acceleration of the liquid due to voidage changes. Eq 2 must be augmented to include terms for these, and the resulting expression for the void fraction is

$$\epsilon = \frac{dz}{dh} + \frac{U_l^2}{g} \frac{d}{dz} \left(\frac{1}{1 - \epsilon} \right) + \frac{4 \tau_w}{\rho_l D_c g} \quad (4)$$

where U_l is the liquid superficial velocity, τ_w is the wall shear stress and D_c is the column diameter. Eq. 4 constitutes an implicit expression for the overall void fraction. In addition its solution requires some method of determining the wall shear stress for a given flow condition. Generally this is done in terms of the friction factor, f . For single phase flows f is obtained from the charts as a function of the Reynolds number. The same approach has been adopted

by Hills with the assumption of a pseudohomogeneous two phase mixture for the dispersion. Merchuk and Stein (1981) use the correlation of Nassos and Bankoff (1967) for the friction factor expressed in terms of the Reynolds number defined for two phase flow.

Fig. 1 illustrates the principle of the differential pressure gradient method. In general, when this technique is adopted the terms corresponding to the friction and the acceleration effects are usually neglected. The lower the gas flow rate, the better this approximation is. One can also use this method to obtain an averaged void fraction in sections of the column by the measurements taken by a series of pressure taps situated along the column height (Prakash and Briens, 1990, Reilly et. al., 1986, Merchuk, 1986, Nicol and Davidson, 1988). The phase fraction obtained is an average over the volume of the dispersion located between

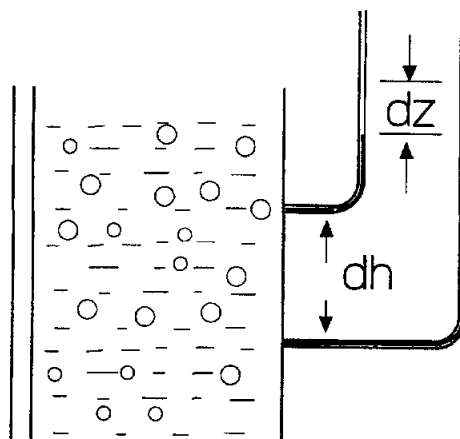


Figure 1: Principle for holdup measurements by means of differential pressure.

the corresponding pressure taps. The method is fairly simple to work with and is applicable to systems at high temperatures and pressures. When one of the phases is a solid, the other being a gas or liquid, then it is necessary to take precautions against clogging of the taps by means of filters or by flushing the taps by a low flow rate of gas. In three phase systems one must also know the solids concentration as a function of height unless the slurry behaves as a pseudohomogeneous mixture.

The overall void fraction ϵ can also be measured by what is referred to as the quick closing valves technique (Rosehart et. al., 1975, Colombo et. al., 1967, Yamaguchi and Yamazaki, 1982). It is based on isolating the two phase dispersion between two sections of the test loop and then measuring the mass of the continuous phase. This, along with the known geometrical parameters and the pressure in the system, can be used for estimating the

holdup between the two sections. The necessary simultaneous closure of two valves that isolate a section of the system is achieved either by coupling the two valves mechanically by springs or by making use of solenoid valves. Although this can be a very accurate method of obtaining a volume averaged gas holdup, it is mainly applicable to small diameter columns. In larger installations the valves may have considerable time constants for closing. Also this method is mainly employed for gas-liquid systems. It can be also be used for gas-solid and liquid-solid systems with moderate solids loading.

Impedance void meters are based on the principle of measuring the electrical impedance of the two phase dispersion. (Gas (air) and liquid (water) have significantly different electrical conductivity and permittivity, and this technique exploits this difference. The variation in the flow structure is accompanied by a variation in the impedance of the two phase mixture which is measured by metallic electrodes introduced suitably in the flow. The void fraction is then estimated by adopting a relative impedance technique. The system can be implemented so that the impedance is governed by the conductance or the capacitance or both. It has to be noted that the impedance of the mixture that is sensed by the electrodes depends not only on the void fraction but also on its distribution (flow regime). The void fraction is calculated as a function of the dielectric constants of the two media using theoretical relations. A relation that has found wide acceptance for low void fractions is the one derived by Maxwell (1881) shown here

$$\frac{\kappa_m - \kappa_w}{\kappa_m + 2\kappa_w} = \epsilon \frac{\kappa_v - \kappa_w}{\kappa_v + \kappa_w} \quad (5)$$

where κ is the dielectric constant, with subscripts m , w and v signifying the mixture, water and vapor, respectively. Another relation for the slug flow regime is

$$\epsilon = \frac{1 - c/c_0}{1 - \kappa_v/\kappa_w} \quad (6)$$

where c and c_0 represent the capacitance of the mixture and the liquid, respectively. As noted by Cimorelli and Evangelistic (1967) the reliability of all such relations is poor and direct calibration for each flow regime is necessary. The technique can be adopted to provide an average over a volume whose size can be large or small, and, accordingly, the measurement can be considered as global or local.

For three phase systems no single method can provide both the solids and overall gas holdup. The expansion of solids (when larger solid particles are used) due to the fluidization by either liquid or gas can be measured in a manner similar to the bed expansion method discussed earlier. Three phase fluidized bed reactors are generally of the continuous flow

type and hence the initial bed height corresponds to the height of the reactor up to which the solids are filled. The bed is then fluidized by either only the liquid phase or a premixed gas-liquid stream. The operating conditions are generally such that the solids are not carried along by the fluids but rather expand to a certain elevation within the column. Once a steady state of operation is established, there exist two distinct regions in the flow referred to as the three phase region and the freeboard region. The height of the three phase region is measured as the expanded bed height. The overall solids holdup in the three phase region is calculated using the expression :

$$\epsilon_s = \text{Vol. of solids loaded} / \text{Vol. of three phase dispersion}$$

If measurements from another technique, such as the pressure drop method are available, the individual gas and liquid holdups can also be obtained in conjunction with the solids holdup obtained from the bed expansion. Since, solids holdup in such systems is often a function of the vertical position in the column, this technique only provides the overall gas and liquid holdup in the three phase region. In gas-sparged slurries with small solid particles, the liquid-solid slurry is most often approximated as a pseudohomogeneous liquid. An effective density is obtained, and the methods described for gas-liquid systems can then be used.

Recommendation : *For estimation of the overall fractional gas holdup one should measure the pressure profile along the reactor length and provide a rough comparison for it by the bed expansion method. The height of the dispersion in the slurry column can be obtained by a γ -ray densitometer (such as the one already in use at Laporte). Table 1 provides a comparison of the characteristics of the available methods for overall gas holdup measurement.*

2.2 Line Average Measurements based on Radiation Techniques

A holdup measurement that can be considered to be in between the overall and point measurement is based on absorption of radiation. Extensive reviews of this technique have been written by Hewitt (1978), Jones and Delhaye (1976), Banerjee and Lahey (1981), and by Snoek (1988). A good introduction of the principles behind some of the techniques has been presented by Schrock (1969).

In most applications it is the attenuation of the radiation (X-rays or gamma-rays) that serves as the basis for the measurement. The absorption of a narrow beam of radiation of initial intensity I_0 by a homogeneous material with a mass attenuation coefficient μ is expressed as :

Table 1: Utility of Different Techniques for Overall Gas Holdup Measurement

	Bed Expansion Method	Pressure Gradient Method	Quick Closing Valves	Impedance Void Meters
Intrusiveness	1	1	5	3
Applicability in Aqueous Systems	Yes	Yes	Yes	No (Capacitive meters)
Applicability in Hydrocarbon Systems	Yes	Yes	Yes	No (Conductance meters)
Applicability in 3-Phase systems	Yes	Not sufficient [®]	Yes	No
Accuracy	3-4	2-3	1-2*	2
Ease of Use & Adaptability	1	2	5	3
Cost of System	1-2	2-3	4	3

Numbers in table indicate a ranking on a scale of 1 to 5. Rank 1 indicates that the technique is most suitable and rank 5 signifies that the technique is not to be preferred. Ranking for the cost of the system is based on 1 representing the least expensive and 5 representing the most expensive system.

* provided valves close rapidly and simultaneously

[®] an additional measurement by an independent technique is required

$$I = I_0 \exp(-\rho \mu l) \quad (7)$$

where I is the intensity of radiation detected after the beam has traveled a distance l through the absorbing medium. For a mixture of two substances, say a gas and a liquid with an attenuation coefficient μ_g and μ_l and densities ρ_g and ρ_l , respectively, the corresponding relation is

$$I = I_0 \exp[-(\rho_g \mu_g l_g + \rho_l \mu_l l_l)] \quad (8)$$

where l_g and l_l are the path lengths of the beam in the gas and the liquid, respectively. In terms of the measured intensities I_{tp} , I_f and I_{mt} corresponding to the test section with the two-phase mixture, full of liquid and completely empty, respectively, the chordal average void fraction is computed from:

$$\epsilon = \frac{\ln(I_{tp}/I_f)}{\ln(I_{mt}/I_f)} \quad (9)$$

This expression is applicable only in two-phase systems and only when a narrow collimated beam is used in the measurement process. However, many early studies have used what is referred to as the one shot method wherein an uncollimated beam of radiation as wide as the test section of interest is used. Use of Eq. 9 in conjunction with this method is known to result in large errors because of the dependence of the attenuation process on the distribution of the voids i.e., ϵ on the flow regimes.

In principle, the void fraction profile can be determined to a fine detail by having a source emitting a narrow beam of radiation and an opposing detector scan across the cross-section. This yields a series of chordal average measurements. To obtain a distribution of the void fraction across a given cross-section, one would have to obtain a series of such scans at different angular orientations. The process of obtaining the voidage profile from such measurement is commonly referred to as tomography. Figure 2 illustrates the differences in the data collection procedures of densitometry and tomography.

For industrial systems it is convenient and less expensive to obtain a few chordal measurements of the attenuation through the test section. Unfortunately, it is often forgotten that a single line averaged holdup, even if it is obtained across the centerline of the column is not representative of the cross sectional mean. The line averaged holdups along several chordal positions can, however, be used to obtain a cross-sectional mean provided that one assumes that the holdup distribution is axisymmetric. If such an assumption can be made, the radial variation of the holdup and hence the cross-sectional mean can be obtained by

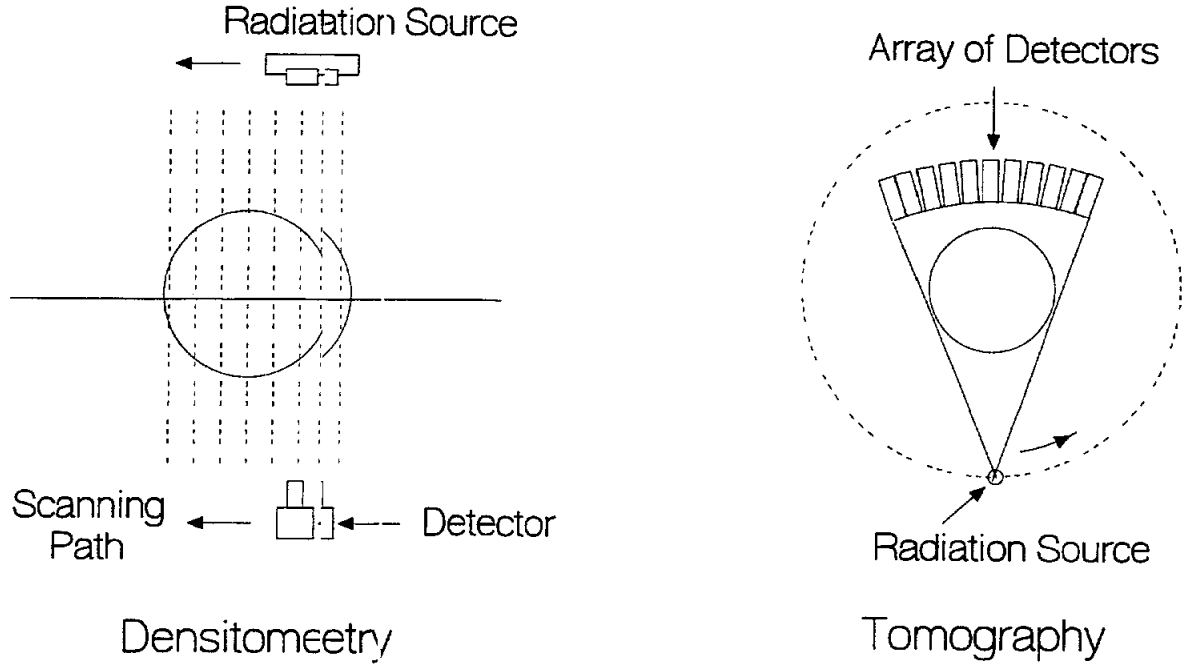


Figure 2: Data collection for densitometry and tomography.

making use of the Abel integral and its inversion.

If $f(r, R)$ is a function of radial position that is nonzero only within a circle of radius R , then its Abel transform is

$$\phi(x, R) = 2 \int_0^{\sqrt{(R^2 - x^2)}} f\left(\sqrt{(x^2 + y^2)}, R\right) dy = 2 \int_x^R \frac{f(r, R) r}{\sqrt{r^2 - x^2}} dr \quad (10)$$

The above is merely the line integral along the ray in the y direction at the position x in the x - y coordinate system. The inversion expressing f in terms of ϕ is

$$f(r, R) = -\frac{1}{\pi} \int_r^R \frac{d\phi/dx}{\sqrt{x^2 - r^2}} dx \quad (11)$$

For our case, the quantity $\phi(x, R)$ corresponds to the quantity $\ln(I_0/I)$ divided by the corresponding chordal length. Many different numerical approaches have been suggested for the implementation of the Abel inversion (Bockasten, 1961, W. L. Barr, 1962, Dong and Kearney, 1991). To illustrate that the centerline averaged holdup is an overestimate of the cross-sectional mean, and that the Abel inversion provides a radial distribution that leads to

the correct cross-sectional mean, the following simulation has been made. An axisymmetric distribution of the holdup is assumed in an air-water bubble column of 19.05 cm in diameter. The assumed radial distribution has the functional form (Kumar, 1994)

$$\epsilon(\xi) = \bar{\epsilon} \frac{m+2}{m} (1 - c\xi^m) \quad (12)$$

where $\bar{\epsilon}$ is the cross-sectional mean holdup, m is the power law exponent and c is a constant that provides for non-zero holdup at the wall of the column. These parameters were obtained from the experimentally determined void fraction distributions using the CT scanner at CREL (Kumar et al., 1995) a schematic of which is shown in Fig.3. The value

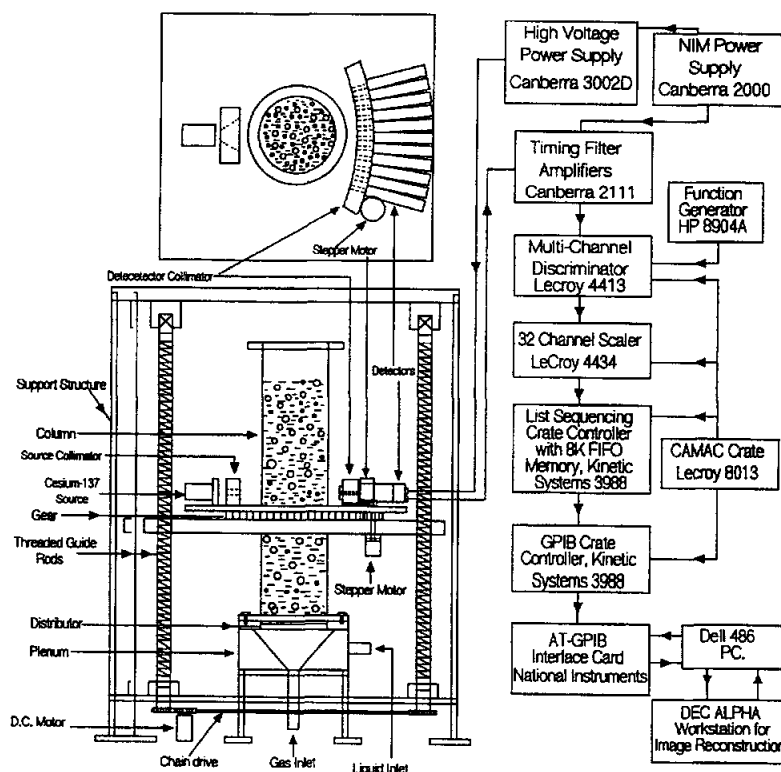


Figure 33: Schematic of the CT scanner at CREL

of the power law exponent m is high for a flat holdup distribution (corresponding to bubbly flow) and decreases for the more parabolic profile that is observed in churn turbulent regime. For a given holdup distribution the line integrals corresponding to the quantity $\int (\mu_{\text{air}} \epsilon + \mu_{\text{water}} (1 - \epsilon)) \rho dl$ are computed. The limits of integration depend on the position of the chord. The negative exponential of this quantity would correspond to the ratio

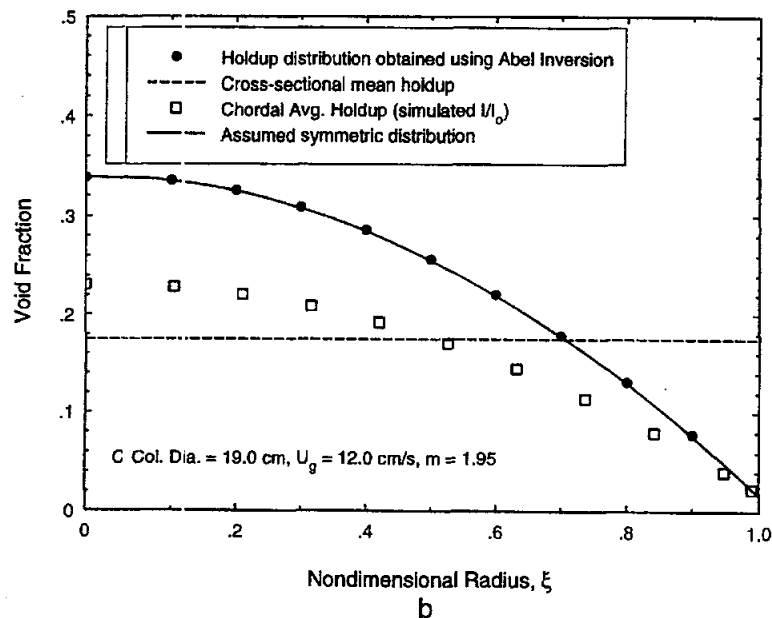
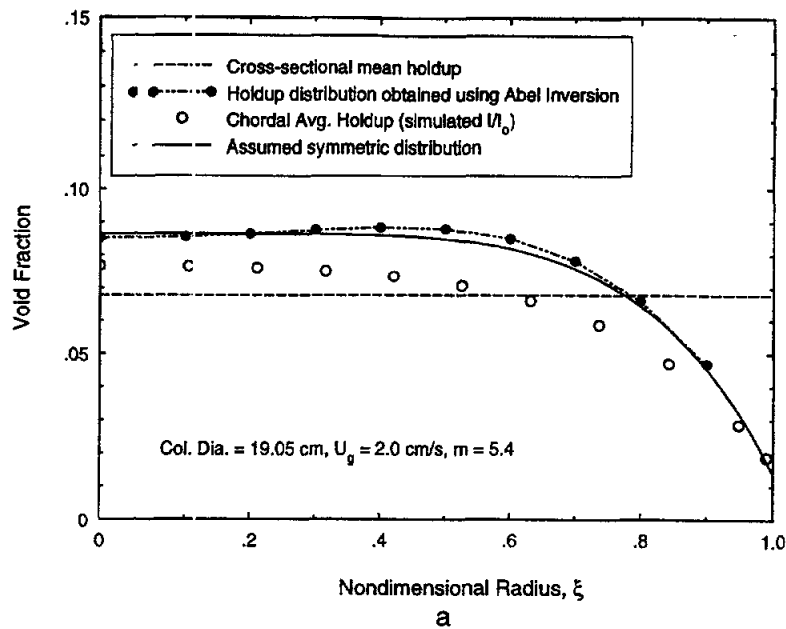


Figure 4: Comparison of assumed distribution, chordal averages and Abel inversion

I/I_0 that is measured in densitometry (as well as CT). Shown in Figures 4 (a) and (b) are the assumed symmetric holdup distributions (similar to experimentally determined holdup profiles in the two flow regimes), and the chordal average holdups that are obtained from the simulated line integrals for two values of m corresponding to bubbly and churn turbulent regimes, respectively. Also shown as a horizontal line for comparison is the cross-sectional mean. The chordal averaged centerline holdup is always higher than the cross-sectional mean. Using the entire set of chordal measurements, the Abel inversion provides a radial distribution that is the same (within numerical error) as the assumed (original) axisymmetric distribution. Abel inversion yields reliable results only when the distribution is axisymmetric. For industrial systems the current measurement practice can be modified to provide

the chordal average measurements at two positions at right angles to each other. An assessment can then be made whether the assumption of axisymmetry is appropriate and if so a cross-sectional mean using Abel inversion can be obtained.

The important advantage of X-ray or γ -ray based techniques is that it can be adopted for studying a wide range of systems including high pressure and high temperature systems. The type of radiation that can be used can be X-rays or γ rays, β rays (applicable only for small test sections) or neutron beams. However, due to their higher penetrating capabilities X-rays and γ rays are the preferred types of radiation. The method lends itself to a certain degree of automation and consequently can be used to provide local information to supplement the global information provided by the bed expansion and the pressure drop method.

The use of densitometry or tomography as described above is not sufficient for three-phase systems for which the pseudohomogeneous approximation for the mixture of solid and liquid phases cannot be made. This is because in this case there are three unknowns viz., the holdups of the three phases, while the attenuation measurement provides only one equation. This along with the overall mass balance is not sufficient to discriminate between all three phases. If the linear attenuation coefficients for the gas, liquid and solid phases in a three-phase system are denoted by μ_g , μ_l and μ_s , respectively, and the corresponding void fractions are denoted as ϵ_g , ϵ_l and ϵ_s respectively, then the transmission ratio for a three phase system can be expressed as :

$$l \ln \frac{I_0}{I} = \int (\mu_g \epsilon_g + \mu_l \epsilon_l + \mu_s \epsilon_s) dl \quad (13)$$

where overall continuity requires

$$\epsilon_g + \epsilon_l + \epsilon_s = 1 \quad (14)$$

All the three quantities ϵ_g , ϵ_l , ϵ_s cannot be obtained by a conventional single energy CT. If measurements taken at two different photon energy levels are available, it is possible to resolve the holdup of the individual phases. This technique, known as dual energy CT, takes advantage of the functional dependence of the attenuation coefficient of a material on the energy of the photons. The solution of the simultaneous equations resulting from the two transmission measurements described by Eq. 13 and Eq. 14 is best accomplished when the radiation at one energy level is primarily attenuated by one phase, and the radiation at the other energy level is attenuated mainly by the other phase. Consequently, the photon energy of the second source needs to be different from the first one by a significant amount in order to provide a unique second measurement for the accurate distinction of the third