APPENDIX D

HYDRODYNAMIC MODEL

TABLE D-1. Governing Equations -- Hydrodynamic Model B for Gas-Liquid-Solid Flow

In the following equations, the tensor is represented as [ ].

D.1 Continuity Equation for Phase k ($= g, l, s$).

$$\frac{\partial}{\partial t}(\epsilon_k \rho_k) + \nabla\cdot(\epsilon_k \rho_k \boldsymbol{v_k}) = 0$$

D.2 Momentum Equations.

a) Liquid Phase (Continuous Phase)

$$\frac{\partial}{\partial t}(\epsilon_l \rho_l \boldsymbol{v_l}) + \nabla\cdot(\epsilon_l \rho_l \boldsymbol{v_l}\boldsymbol{v_l}) = -\nabla p_l + \rho_l \boldsymbol{g} + \sum_{m=g,s} \beta_{lm}(\boldsymbol{v_m} - \boldsymbol{v_l}) + \nabla\cdot[\boldsymbol{\tau_l}]$$

b) Gas / Solid Phases k ($= g, s$) (Dispersed Phases)

$$\frac{\partial}{\partial t}(\epsilon_k \rho_k \boldsymbol{v_k}) + \nabla\cdot(\epsilon_k \rho_k \boldsymbol{v_k}\boldsymbol{v_k}) = -\nabla p_k + \frac{\epsilon_k}{\epsilon_l}\Big(\rho_k - \sum_{m=l,g,s\; m\neq k} \epsilon_m \rho_m\Big)\boldsymbol{g} + \sum_{m=l,g,s\; m\neq k} \beta_{km}(\boldsymbol{v_m} - \boldsymbol{v_k}) + \nabla\cdot[\boldsymbol{\tau_k}]$$

D.3 Constitutive Equations for Stress.

a) Fluid Phase Stress. k ($= l, g$).

$$[\boldsymbol{\tau_k}] = \epsilon_k \mu_k \left([\nabla \boldsymbol{v_k} + (\nabla \boldsymbol{v_k})^T] - \frac{2}{3}\nabla\cdot(\boldsymbol{v_k})\,[\boldsymbol{I}] \right)$$

b) Solid Phase Stress.

$$[\boldsymbol{\tau_s}]=\mu_s[\nabla\boldsymbol{v_s}+(\nabla\boldsymbol{v_s})^T]+(\xi_s-\frac{2}{3}\mu_s)\nabla\cdot(\boldsymbol{v_s})\,[\boldsymbol{I}]$$

D.4.1 Kinetic Theory Model.

a) Fluctuating Energy Θ_s (= ⅓ < C^2 >) Equation,

$$\frac{3}{2}\left(\frac{\nabla}{\nabla t}(\epsilon_s\rho_s\Theta_s)+\nabla\cdot(\epsilon_s\rho_s\Theta_s\boldsymbol{v_s})\right)=-p_s\nabla\cdot\boldsymbol{v_s}+\Phi_k-\nabla\cdot\boldsymbol{q_s}-\gamma_s-3\beta_{1s}\Theta_s$$

b) Collision Energy Dissipation Rate,

$$\gamma_s=3(1-e^2)\,\epsilon_s^2\rho_s d_s g_0\Theta_s\left(\frac{4}{d_k}\left(\frac{\Theta_s}{\pi}\right)^{\frac{1}{2}}-\nabla\cdot\boldsymbol{v_s}\right)$$

c) Flux of Fluctuating Energy,

$$\boldsymbol{q_s}=-\kappa_s\nabla\Theta_s$$

d) Conductivity of Fluctuating Energy,

$$\kappa_s=\frac{2\kappa_{dil}}{(1+e)\,g_0}\left[1+\frac{6}{5}(1+e)\,g_0\epsilon_s\right]^2+2\epsilon_s^2\rho_s d_s(1+e)\,g_0\left(\frac{\Theta_s}{\pi}\right)^{\frac{1}{2}}$$

e) Dilute Phase ("Eddy Type") Granular Conductivity,

$$\kappa_{dil}=\frac{25\sqrt{\pi}}{128}\rho_s d_s\Theta_s^{\frac{1}{2}}$$

f) Solid Phase Shear Viscosity,

$$\mu_s=\frac{2\mu_{dil}}{(1+e)\,g_0}\left[1+\frac{4}{5}(1+e)\,g_0\epsilon_s\right]^2+\frac{4}{5}\epsilon_s^2\rho_s d_s(1+e)\,g_0\left(\frac{\Theta_s}{\pi}\right)^{\frac{1}{2}}$$

g) Solid Phase Bulk Viscosity,

$$\xi_s = \frac{4}{3}\epsilon_s^2 \rho_s d_s (1+e) g_o \left(\frac{\Theta_s}{\pi}\right)^{\frac{1}{2}}$$

h) Dilute Phase Solids Viscosity,

$$\mu_{dil} = \frac{5\sqrt{\pi}}{96} \rho_s d_s \Theta_s^{\frac{1}{2}}$$

i) Solid Phase Pressure

$$p_s = \epsilon_s \rho_s \Theta_s [1 + 2(1+e)\epsilon_s g_0]$$

j) Radial Distribution Function,

$$g_0 = \left[1 - \left(\frac{\epsilon_s}{\epsilon_{s,\max}}\right)^{\frac{1}{3}}\right]^{-1}$$

k) Energy Dissipation Rate

$$\Phi_s = [\tau_s] : \nabla \mathbf{v}_s$$

D.4.2 Empirical Solids Viscosity and Stress Model.

$$\nabla p_k = G(\epsilon_k) \nabla \epsilon_k$$

$$\xi_s = 0$$

$$\mu_s = 5\epsilon_s \quad (poises) \qquad (example)$$

D.5 Intra--Phase Drag Coefficients. *k (= g, s)*

a) for $\epsilon_l < 0.8$, (based on Ergun equation)

$$\beta_{lk} = \beta_{kl} = 150 \frac{(1-\epsilon_l)\epsilon_k \mu_l}{(\epsilon_l d_k \psi_k)^2} + 1.75 \frac{\rho_l \epsilon_k |\mathbf{v}_l - \mathbf{v}_k|}{\epsilon_l d_k \psi_k}$$

b) for $\epsilon_l \geq 0.8$, (based on empirical correlation)

$$\beta_{1k}=\beta_{k1}=\frac{3}{4}C_D\frac{\epsilon_k\rho_1|\mathbf{v}_1-\mathbf{v}_k|}{d_k\psi_k}\epsilon_1^{-2.65}$$

where,

$$C_D=\begin{cases}\frac{24}{Re_k}(1+0.15Re_k^{0.687}) & for\ Re_k<1000\\ 0.44 & for\ Re_k\geq 1000\end{cases}$$

$$Re_k=\frac{\epsilon_1\rho_1|\mathbf{v}_1-\mathbf{v}_k|d_k\psi_k}{\mu_1}$$

D.6 Particle--Particle Drag Coefficients. *k, m (= l, g, s)*

$$\beta_{km\ k\neq m}=\frac{\alpha(1+e)\epsilon_k\rho_k\epsilon_m\rho_m(d_k+d_m)^2\left[1+3\left(\frac{\epsilon_{km}}{\epsilon_k+\epsilon_m}\right)^{\frac{1}{3}}\right]}{2\epsilon_f(\rho_k d_k^3+\rho_m d_m^3)\left[\left(\frac{\epsilon_{km}}{\epsilon_k+\epsilon_m}\right)^{\frac{1}{3}}-1\right]}|\mathbf{v}_k-\mathbf{v}_m|$$

where

$$\epsilon_{km}=\begin{cases}[(\phi_k-\phi_m)+(1-a)(1-\phi_k)\phi_m][\phi_k+(1-\phi_m)\phi_k]\frac{X_k}{\phi_k}+\phi_m & for\ X_k\leq X_{k,o}\\ (1-a)[\phi_k+(1-\phi_k)\phi_m](1-X_k)+\phi_k & for\ X_k\geq X_{k,o}\end{cases}$$

$$a=\sqrt{\frac{d_m}{d_k}}\qquad (d_k\geq d_m)$$

$$X_k=\frac{\epsilon_k}{\epsilon_k+\epsilon_m}$$

$$X_{k,o}=\frac{\phi_k}{[\phi_k+(1-\phi_k)\phi_m]}$$

D.7 Definitions.

$$\epsilon_l+\epsilon_g+\epsilon_s=1$$

The gas, liquid and solid phases are assumed to be incompressible and constant densities are used. The constitutive relations in kinetic theory model were derived by Gidaspow (1994). The constitutive relations may be easily changed by some program modification.