MODELLING

A. A BUBBLE-SLUG FLOW MODEL

MODEL DEVELOPMENT

The basis of the physical model developed in the subsequent paragraphs is founded on the following assumptions. These include a system of fully developed, axially symmetric, steady state, two-phase flow at isothermal and low pressure conditions. Also, it is assumed there is equilibrium between phases (i.e. no mass transfer), both the coalescence and breakage rates are equivalent, and the gas phase undergoes negligible expansion for the short rise distance of one bubble-slug unit length.

Actual observations of this flow pattern would show highly fluctuating, spatial distribution of the gas and liquid phases within the column. However, based on the characteristic hydrodynamics of this flow pattern, one can generalize as to the physical geometry which prevails. This consists of the tendency for the agglomerated, large spherical cap shaped bubbles to rise quickly, up the center of the column, through a nearly uniform dispersion of smaller spherical bubbles in a flowing liquid. Therefore, it is convenient to view the bubble-slug flow pattern for an idealized geometry, as presented in Figure 1, which shows a bubble-slug unit as 'frozen' for an instant in time. The results of this model can be considered as giving time-area averaged values for the parameters involved.

The bubble-slug unit, as depicted in Figure 1, consists of two segregated sections, referred to as the large bubble region, and the liquid slug region. The large bubble region consists of the large spherical cap shaped bubble, formed by coalescence and agglomeration of the smaller spherical bubbles, and the surrounding liquid which contains some of the small spherical bubbles. The liquid slug region also contains these small spherical bubbles.

Paralleling the development of Fernandes et. al., (36) this model development begins with defining the volume average void of the bubble-slug unit as follows:

$$\alpha_{BSU} = \frac{V_G}{V_{BSU}} \tag{1}$$

where VG is the total volume of the gas phase present in the bubble-slug unit, and VBSU is the volume of the bubble-slug unit itself, which is equal to:

$$= \ell_{BSU} A = (\ell_{LB} + \ell_{LS}) A \tag{2}$$

The total volume of the gas present in the bubble-slug unit is equal to the following:

$$V_G = V_{GLB} + V_{GSB} + V_{GLS} \tag{3}$$

where VGLB is the volume of gas in the large bubble, VGSB is the volume of gas present as small bubbles in the large bubble region, and VGLS is the volume of gas present as small bubbles in the liquid slug region.

If it can safely be assumed that the small spherical bubbles are uniformly distributed throughout both the large bubble and liquid slug regions, one can write:

$$V_{GLB} = \ell_{LB} A_{GLB} \tag{4a}$$

$$V_{GSB} = \ell_{LB} A_{GSB} \tag{4b}$$

$$V_{GLS} = \ell_{LS} A_{GLS} \tag{4c}$$

where these areas are the effective fractional cross-sections occupied by the gas in each designated region. Substituting these relations into equation (3) results in the following expression:

$$V_G = \ell_{LB} A_{GLB} + \ell_{LB} A_{GSB} + \ell_{LS} A_{GLS} \tag{5}$$

Returning to the definition of the total void fraction of the bubble-slug unit, and utilizing the above relations gives:

$$\alpha_{BSU} = \frac{V_G}{V_{BSU}} = \frac{\ell_{LB} A_{GLB} + \ell_{LB} A_{GSB} + \ell_{LS} \cdot A_{GLS}}{\ell_{BSU} A} \tag{6}$$

Let us define the relative length of the large bubble region to the length of the bubble-slug unit as follows:

$$\beta = \frac{\ell_{LB}}{\ell_{BSU}} \tag{7}$$

Following our assumption of uniformly axial distribution of the small spherical bubbles, one can approximately equate volume void fractions with area average void fractions, per bubble-slug unit length, as follows:

$$\alpha_{LB} = \frac{V_{GLB}}{V_{BSU}} = \frac{A_{GLB}}{A} \tag{8a}$$

$$\alpha_{SB} = \frac{V_{GSB}}{V_{BSU}} = \frac{A_{GSB}}{A} \tag{8b}$$

$$\alpha_{LS} = \frac{V_{GLS}}{V_{BSU}} = \frac{A_{GLS}}{A} \tag{8c}$$

Substituting equations (7) and (8a,b,c) into (6) and slightly rearranging, gives the expression for the total bubble-slug unit void fraction from the relative contribution of each region.

$$\alpha_{BSU} = \alpha_{LB}\,\beta + \alpha_{SB}\,\beta + \alpha_{LS}\,(1 - \beta) \tag{9}$$

CONTINUITY BALANCES

Restricting our attention to a single bubble-slug unit, the time required for the bubble-slug unit to pass a fixed reference plane can be expressed as:

$$\Delta t_{BSU} = \frac{\ell_{BSU}}{U_{BSU}} \tag{10}$$

Likewise, the time intervals for the separate large bubble and liquid slug portions to pass the reference plane can be expressed, respectively, as:

$$\Delta t_{LB} = \frac{\ell_{LB}}{U_{BSU}} \tag{11a}$$

$$\Delta t_{LS} = \frac{\ell_{LS}}{U_{BSU}} \quad (11b)$$

During the same time interval for the large bubble region, the surrounding dispersed small spherical bubbles also pass the reference plane, such that:

$$\Delta t_{SB} = \frac{\ell_{LB}}{U_{BSU}} \quad (11c)$$

In each case, UBSU is the average translational velocity of the bubble-slug unit (which can be viewed as the average rise velocity of all the gas bubbles present in the unit).

On the basis of assuming an incompressible liquid and negligible expansion of the large gas bubble for the rise distance of one bubble-slug unit length, one can safely assume that volume balances are equivalent to mass balances (i.e. constant density). The volume of gas transported past this reference plane in the large bubble is thus:

$$V_{GLB} = U_{GLB}\,\alpha_{LB}\,A\,\Delta t_{LB} = U_{GLB}\,\alpha_{LB}\,A\,\frac{\ell_{LB}}{U_{BSU}} \quad (12)$$

The volume of gas transported as small spherical bubbles associated with the large bubble region is:

$$V_{GSB} = U_{GSB}\,\alpha_{SB}\,A\,\Delta t_{LB} = U_{GSB}\,\alpha_{SB}\,A\,\frac{\ell_{LB}}{U_{BSU}} \quad (13)$$

And the volume of gas transported as small spherical bubbles associated with the liquid slug region is:

$$V_{GLS} = U_{GLS}\,\alpha_{LS}\,A\,\Delta t_{LS} = U_{GLS}\,\alpha_{LS}\,A\,\frac{\ell_{LS}}{U_{BSU}} \tag{14}$$

The total gas volumetric throughput can therefore be expressed by:

$$Q_G\,\Delta t_{BSU} = U_{SG}\,A\,\frac{\ell_{BSU}}{U_{BSU}} = U_{SG}\,A\,\frac{(\ell_{LB} + \ell_{LS})}{U_{BSU}} \tag{15}$$

or alternately

$$U_{SG}\,A\,\frac{\ell_{BSU}}{U_{BSU}} = V_{GLB} + V_{GSB} + V_{GLS} \tag{16}$$

Substituting equations (12), (13), (14) into (16) will give:

$$U_{SG}\,A\,\frac{\ell_{BSU}}{U_{BSU}} = U_{GLB}\,\alpha_{LB}\,A\,\frac{\ell_{LB}}{U_{BSU}} + U_{GSB}\,\alpha_{SB}\,A\,\frac{\ell_{LB}}{U_{BSU}} + U_{GLS}\,\alpha_{LS}\,A\,\frac{\ell_{LS}}{U_{BSU}} \tag{17}$$

Simplifying, remembering the relation given by equation (7), results in the following:

$$U_{SG} = U_{GLB}\,\alpha_{LB}\,\beta + U_{GSB}\,\alpha_{SB}\,\beta + U_{GLS}\,\alpha_{LS}\,(1 - \beta) \tag{18}$$

Similarly, for the liquid phase, the volume of liquid associated with the large bubble region which passes the reference plane is given by:

$$\begin{aligned} V_{LLB} &= U_{LLB}\,(1 - \alpha_{LB} - \alpha_{SB})\,A\,\Delta t_{LB} \\ &= U_{LLB}\,(1 - \alpha_{LB} - \alpha_{SB})\,A\,\frac{\ell_{LB}}{U_{BSU}} \end{aligned} \tag{19}$$

And for the volume of liquid in the liquid slug region:

$$V_{LLS} = U_{LLS}\ (1 - \alpha_{LS})\ A\ \Delta t_{LS} = U_{LLS}\ (1 - \alpha_{LS})\ A\ \frac{\ell_{LS}}{U_{BSU}} \qquad (20)$$

The total volumetric liquid rate is thus

$$Q_L\ \Delta t_{BSU} = U_{SL}\ A\ \frac{\ell_{BSU}}{U_{BSU}} = U_{SL}\ A\ \frac{(\ell_{LB} + \ell_{LS})}{U_{BSU}} \qquad (21)$$

or alternately:

$$U_{SL}\ A\ \frac{\ell_{BSU}}{U_{BSU}} = V_{LLB} + V_{LLS} \qquad (22)$$

Substituting equations (19) and (20) into (22) gives:

$$U_{SL}\ A\ \frac{\ell_{BSU}}{U_{BSU}} = U_{LLB}\ (1 - \alpha_{LB} - \alpha_{LS})\ A\ \frac{\ell_{LB}}{U_{BSU}} + U_{LLS}\ (1 - \alpha_{LS})\ A\ \frac{\ell_{LS}}{U_{BSU}} \qquad (23)$$

Simplifying, again remembering equation (7), gives:

$$U_{SL} = U_{LLB}\ (1 - \alpha_{LB} - \alpha_{SB})\ \beta + U_{LLS}\ (1 - \alpha_{LS})(1 - \beta) \qquad (24)$$

Equations (18) and (24) give the expressions for the continuity balances of each phase, relative to their respective superficial velocity. A second pair of continuity relations can be derived, relative to the average rise velocity of the bubble-slug unit as a whole. For the gas phase, it can be shown that:

$$(U_{BSU} - U_{GLB})\,\alpha_{LB} + (U_{BSU} - U_{GSB})\,\alpha_{SB} = (U_{BSU} - U_{GLS})\,\alpha_{LS} \tag{25}$$

And similarly, for the liquid phase:

$$(U_{BSU} - U_{LLB})(1 - \alpha_{LB} - \alpha_{SB}) = (U_{BSU} - U_{LLS})(1 - \alpha_{LS}) \tag{26}$$

AVERAGE LIQUID VELOCITY PROFILE

Walter and Blanch (135) reported on the liquid circulation patterns and their effect upon the gas holdup using both microscopic and macroscopic balances. They began their development with deriving two forms of the average liquid velocity profiles from microscopic momentum balances. The first form was for what they termed 'turbulent flow, which provided for an assumed slippage at the column wall, while the second form of liquid velocity profile was for a no slip condition at the column wall which they termed 'slow flow.

They began with the equation of motion for axially symmetric two phase flow in a cylinder as follows:

$$-\frac{1}{r}\frac{\partial}{\partial r}(r\tau) = \frac{\partial P}{\partial z} + (1 - \alpha_G)\rho_L\, g \tag{27}$$

where is indicative of the liquid rheology. The integrated form of this equation over the column cross-section is

$$\frac{dP}{dz} = \frac{2}{R}\tau_W + (1 - \bar{\alpha}_G)\rho_L g \tag{28}$$

At this point, Walter and Blanch (135) substituted the gas holdup radial profile obtained by Ueyama and Miyauchi (163), which is given below, into the above equation

$$\alpha_G = \bar{\alpha}_G\left(\frac{N + 2}{N}\right)\left[1 - \left(\frac{r}{R}\right)^N\right] \tag{29}$$

The parameter N is a constant to account for the shape of the gas holdup profile. Ueyama and Miyauchi (163) reported this value falls within the range from 1.8 to 2.0 for the air-water system.

Walter and Blanch (135) stated that if the eddy or bulk viscosity can be assumed constant, the resultant liquid velocity profile has the form:

$$\frac{U_L}{U_{max}} = A'\left(\frac{r}{R}\right)^2 - B'\left(\frac{r}{R}\right)^{N+2} + C' \ln\left(\frac{r}{R}\right) + D' \tag{30}$$

or in dimensionless form:

$$\underline{U}_L = A'\underline{r}^2 - B'\underline{r}^{N+2} + C' \ln \underline{r} + D' \tag{31}$$

Upon integrating the above expression, Walter and Blanch (135) used boundary conditions corresponding to no net flow of the liquid. This study, however, will incorporate a provision for a flowing liquid phase whose net flow is equivalent to the area average velocity (i.e. superficial velocity). Also, of primary interest is the case of 'slow flow' with no slip at the column wall. This is because it is believed that in columns of large diameter, the diameter will be of such a magnitude relative to the thickness of the down flowing liquid annular region, that laminar boundary layers will prevail and therefore no slip will occur at the wall. The boundary conditions for this integration are listed below.

$$\text{B. C. 1:} \quad \frac{dU}{dr} = 0 \quad @ \quad r = 0 \tag{32}$$

$$\text{B. C. 2:} \quad U_L = U_{max} \quad @ \quad r = 0 \tag{33}$$

$$\text{B. C. 3:} \quad \frac{\int_0^R U(r)\, r\, dr}{\int_0^R r\, dr} = U_{SL} \tag{34}$$

$$\text{B. C. 4:} \quad U_L = 0 \quad @ \quad r = R \tag{35}$$

The third boundary condition provides for net flow of the liquid equal to the superficial velocity, while the fourth is the case for no slip at the column wall. Upon integrating equation (31), boundary condition 1 stipulates that C' must equal zero, which with boundary condition 2, determines that D' is equal to one. The integrated form of boundary condition 3 results in:

$$2U_{max}\left[\frac{A'}{4}\left(\frac{r}{R}\right)^4 - \frac{B'}{N+4}\left(\frac{r}{R}\right)^{N+4} + \frac{D'}{2}\left(\frac{r}{R}\right)^2\right] = U_{SL} \tag{36}$$

Since it is known D'=1, the above relation, with boundary condition 4, can be used to solve for the coefficients A' and B', whose resultant expressions are given below.

$$A' = \left(\frac{N+4}{N}\right)\left(\frac{2U_{SL}}{U_{max}} - 1\right) - 1 \tag{37}$$

$$B' = \left(\frac{N+4}{N}\right)\left(\frac{2U_{SL}}{U_{max}} - 1\right) \tag{38}$$

Substituting these expressions for the coefficients into equation (30) and rearranging gives the expression for the average liquid velocity profile with a net flow equivalent to the superficial velocity:

$$\frac{U_L}{U_{max}} = 1-\left(\frac{r}{R}\right)^2 + \left(\frac{N+4}{N}\right)\left(1-\frac{2U_{SL}}{U_{max}}\right)\left[\left(\frac{r}{R}\right)^{N+2} - \left(\frac{r}{R}\right)^2\right] \tag{39}$$

Which for the air-water system (i.e. N=2) is:

$$\frac{U_I}{U_{max}} = 1-\left(4-\frac{6U_{SL}}{U_{max}}\right)\left(\frac{r}{R}\right)^2 + \left(3-\frac{6U_{SL}}{U_{max}}\right)\left(\frac{r}{R}\right)^4 \tag{40}$$

The above equation can be treated as a quadratic by letting $X=(r/R)^2$, to solve for the transiton point where the liquid axial velocity is equal to zero. Also, one can determine from the quadratic solution expression, the limitation for the quotient USL/Umax. The lower limit on this relation is that Umax must be greater than three times USL.

Upon setting USL equal to zero in equation (39), one quadratic solution results at r/R=0.577, which is the exact root of Walter and Blanch's velocity profile for the case of no net liquid flow. Liquid velocity profiles for both the cases of no net liquid flow and a positive net flow equal to the superficial velocity of 0.05 m/sec are shown in Figure 2 for comparison. It can readily be seen that with increasing superficial liquid velocity, the transition point progresses outward, radially toward the wall, and thus the down flowing liquid annular region decreases.

Of specific interest, is the average axial liquid velocity in the annular area of the large bubble region. This liquid velocity value can be determined by performing an area averaging integration on equation (40), for the air-water system, within the appropriate limits. The general form of this type of integration is given below:

$$\bar{U}_L = \frac{\int_{r_1}^{r_2} U(r)\, r dr}{\int_{r_1}^{r_2} r dr} \tag{41}$$

For our specific application using equation (40) is:

$$U_{LLB} = U_{max} \int_{r_b}^{R} \left[1 - \left(4 - \frac{6U_{SL}}{U_{max}}\right)\left(\frac{r}{R}\right)^2 + \left(3 - \frac{6U_{SL}}{U_{max}}\right)\left(\frac{r}{R}\right)^4 \right] r\, dr \tag{42}$$

where r_b is the radius of the large bubble, and R is the column radius. Performing the above integration results in the following:

$$U_{LLB} = \frac{U_{max}}{1 - (r_b/R)^2} \left[1 - (r_b/R)^2 - \frac{1}{2}\left(4 - \frac{6U_{SL}}{U_{max}}\right)(1 - (r_b/R)^4) + \frac{1}{3}\left(3 - \frac{6U_{SL}}{U_{max}}\right)(1 - (r_b/R)^6) \right] \tag{43}$$

However, the large bubble radius which would prevail for a given flow condition is not generally known. Returning to the definition of the volumetric void fraction, one can show the following holds true for either a spherical or a hemi-spherical shaped bubble.

$$\alpha_{LB} = \frac{2}{3}\left(\frac{r_b}{R}\right)^2 \tag{44}$$

or

$$\frac{r_b}{R} = \left(\frac{2}{3}\alpha_{LB}\right)^{1/2} \tag{45}$$

Substituting the above relation into equation (43) gives:

$$U_{LLB} = \frac{U_{max}}{1 - (\frac{3}{2}\alpha_{LB})} \left[1 - (\frac{3}{2}\alpha_{LB}) - \frac{1}{2}(4 - \frac{6U_{SL}}{U_{max}})(1 - (\frac{3}{2}\alpha_{LB})^2) + \frac{1}{3}(3 - \frac{6U_{SL}}{U_{max}})(1 - (\frac{3}{2}\alpha_{LB})^3) \right] \tag{46}$$

The above expression contains the variable, Umax, which is the center line liquid velocity value. This quantity depends, to a great extent, upon the liquid circulation strength, which is induced by the rapidly rising large gas bubbles. Since the size and frequency of these large bubbles is dependent upon the gas rate, then the gas rate has a direct influence upon the liquid circulation strength, and thus an effect upon Umax.

In large diameter columns, it is believed one can roughly approximate the liquid center line velocity by the difference between the average translational velocity of the bubble-slug unit and the average slip velocity as follows:

$$U_{max} = U_{BSU} - U'_S \tag{47}$$

where the slip velocity, Us', is based on the average total gas void fraction of the bubble-slug unit, as given by:

$$U'_S = \frac{U_{SG}}{\alpha'_{BSU}} - \frac{U_{SL}}{1 - \alpha'_{BSU}} \tag{48}$$

SMALL BUBBLE RISE VELOCITY

It was assumed the small spherical bubbles present in both the liquid slug and the region surrounding the large bubble were uniformly distributed. This distribution can be approximated by cross-sectional average values such that one can write:

$$U_{GSB} = U_{LLB} + U_{b\infty} \quad (49)$$

$$U_{GLS} = U_{LLS} + U_{b\infty} \quad (50)$$

where $U_{b\infty}$ is the rise velocity of the gas bubble due to the bouyancy.

Zuber and Hench (166) modified the relation for the rise velocity of a single bubble by Harmathy to account for the influence of the neighboring bubbles. The hinderance on the rise velocity of the single bubble by the swarm of bubbles, as correlated by Zuber and Hench, is given by the following expression.

$$U_{b\infty} = 1.41 \left[\frac{g\,\sigma\,(\rho_L - \rho_G)}{\rho_L^2} \right]^{1/4} (1 - \alpha_G)^K \quad (51)$$

where K takes on values from 3.0 for very minute bubbles to 1.5 for somewhat larger bubbles. For the size bubbles relevant to this study, K is equal to 1.5.

The average gas void fraction of the small bubbles in the large bubble region is assumed to be equivalent to the average void fraction of the small bubbles in the liquid slug region, however, one must account for the void volume occupied by the large bubble. Therefore, for the void fraction of the small bubbles in the large bubble region, it can be shown that the following is true:

$$[illegible] = \alpha_{LS}\,(1 - \alpha_{LB}) \quad (52)$$

Since these small spherical bubbles have been identified and associated with segregated regions, each average bubble rise velocity must reflect their respective average regional void fractions. Therefore, the absolute average rise velocity of the small spherical bubbles, present in a swarm of bubbles, in both the large bubble and liquid slug regions are, respectively, given by the following two equations:

$$U_{GSB} = U_{LLB} + 1.41\left[\frac{g\sigma(\rho_L - \rho_G)}{\rho_L^2}\right]^{1/4}(1 - \alpha_{SB})^{3/2} \qquad (53)$$

$$U_{GLS} = U_{LLS} + 1.41\left[\frac{g\sigma(\rho_L - \rho_G)}{\rho_L^2}\right]^{1/4}(1 - \alpha_{LS})^{3/2} \qquad (54)$$

AVERAGE BUBBLE-SLUG UNIT VELOCITY

At moderate superficial gas rates, within the bubble-slug flow pattern range, both bubble coalescence and breakage are prevalent, and two average bubble sizes coexist. The smaller bubbles are generally spherical to ellipsoidal in shape and rise vertically in a spiraling, zig-zag path. The agglomerated larger, spherical cap shaped bubbles tend to rise almost in a rectilinear fashion, at a velocity independent of bubble size.

The larger spherical cap shaped bubbles rise at a velocity much greater than that of the smaller spherical bubbles. They tend to sweep through this dispersion, causing turbulent eddies by their wakes, resulting in a recirculation of the iquid phase. This recirculation of the liquid can entrain some of the small spherical bubbles, if the liquid velocity is greater than the rise velocity of the small bubbles.

The relation recommended by Zuber et al. (167) for the rise velocity of bubbles, for the case of bubble size independence, is given by:

$$U_{b\infty} = 1.41\left[\frac{g\,\sigma\,(\rho_L - \rho_G)}{\rho_L^{\,2}}\right]^{1/4} \tag{55}$$

To account for a flowing liquid system, Govier and Aziz proposed addition of the following term to the above relation, with an appropriate coefficient.

$$U_M = U_{SG} + U_{SL} \tag{56}$$

The resultant combined expression can be used to approximate the average translational velocity for a bubble-slug unit, as given below:

$$U_{BSU} = 1.25\,U_M + 1.41\left[\frac{g\,\sigma\,(\rho_L - \rho_G)}{\rho_L^{\,2}}\right]^{1/4} \tag{57}$$

MODEL SOLUTION

At present, the model for the bubble-slug flow pattern consists of 12 basic equations with 13 unknowns. These unknowns represent the major variables and parameters of interest and are listed below.

BSU, LB, SB, LS,

UGLB, UGSB, UGLS

ULLB, ULLS, UBSU

Umax, Us

Like the slug flow model of Fernandes et. al., this equation network can be closed for solution by assuming a value for the average gas void fraction of the liquid slug region which corresponds to the void fraction at the bubble to bubble-slug transition. A solution algorithm similar to that used for the slug flow model can then be followed:

The value of the average liquid slug void fraction which corresponds to the transitional bubble to slug flow void fraction has been reported to be approximately 0.25 (162). However, Beinhauer (19) who used x-ray absorption techniques to measure the relative gas holdup in bubble columns, was able to differentiate the relative contributions between the small and larger gas bubbles to the overall total gas holdup present in the column. He presented his results graphically which is reproduced and shown as Figure 3.

This figure indicates that the relative gas holdup of the small bubbles increases with increasing gas rate to a maximum of approximately 0.22 at a superficial gas vleocity of 0.06 m/sec. The total gas holdup is about 0.25 for the range of 0.055 to 0.08 m/sec. The small bubbles gas holdup then decreases to a somewhat constant value of approximatey 0.19 for USG 0.09 m/sec.

From the results of Beinhauer (9) for the gas range of interest (i.e. the transition from bubble flow to bubble-slug flow, USG=0.04-0.12 m/sec), the average relative gas holdup contribution from the small bubbles is taken to equal 0.20. This study will assume this value for the average gas void fraction from the small bubbles in the liquid slug portion and use it throughout the testing and evaluation of the proposed bubble-slug flow model.

An auxillary equation is now introduced to facilitate the convergence to a solution. This auxillary equation will aid in determining the average total gas void fraction of the bubble-slug unit for use in estimating the average slip velocity determined by equation (48). It can be shown that the following relation holds true and it was found to correlate well upon convergence.

$$\alpha'_{BSU} = (1 - \alpha_{LS})\, \beta\, \alpha_{LB} + \alpha_{LS} \qquad (58)$$

All of the equations which make up this bubble-slug model are summarized in Table 1. The finalized solution algorithm is outlined in Table 2. The convergence method used, was a Bisection method, also known as the Interval Search and Half method. Using the solution algorithm as outlined in Table 2, convergence usually occurred within 45 iterations from the initial assumed value of 1.0 for the large bubble void fraction, TB, and with an initial search interval of 0.05.

A check on the uniqueness of solution was performed similar to that used during the review of the slug flow model. This was accomplished by iteration of the assumed value for the large bubble void fraction and determining the value as calculated by the model, as per the solution algorithm procedure. Figure 4 shows, for a particular set of parameters, the plot of assumed large bubble void fraction values versus calcualted values of the large bubble void fraction from the bubble-slug flow model and does, in fact, indicate a unique solution point for this model.

Table 1

BUBBLE-SLUG MODEL

Equation Network

1. Average total void fraction of bubble-slug unit

$$\alpha_{BSU} = \alpha_{LB}\,\beta + \alpha_{SB}\,\beta + \alpha_{LS}\,(1 - \beta) \tag{9}$$

2. Mass balance for gas phase

$$U_{SG} = U_{GLB}\,\alpha_{LB}\,\beta + U_{GSB}\,\alpha_{SB}\,\beta + U_{GLS}\,\alpha_{LS}\,(1 - \beta) \tag{18}$$

3. Mass balance for liquid phase

$$U_{SL} = U_{LLB}\,(1 - \alpha_{LB} - \alpha_{SB})\,\beta + U_{LLS}\,(1 - \alpha_{LS})(1 - \beta) \tag{24}$$

4. Gas flow relative to average velocity of bubble-slug

$$(U_{BSU} - U_{GLB})\,\alpha_{LB} + (U_{BSU} - U_{GSB})\,\alpha_{SB} = (U_{BSU} - U_{GLS})\,\alpha_{LS} \tag{25}$$

5. Liquid flow relative to average velocity of bubble-slug

$$(U_{BSU} - U_{LLB})(1 - \alpha_{LB} - \alpha_{SB}) = (U_{BSU} - U_{LLS})(1 - \alpha_{LS}) \tag{26}$$

6. Rise velocity of small bubbles in large bubble region

$$U_{GSB} = U_{LLB} + 1.41\left[\frac{g\,\sigma\,(\rho_L - \rho_G)}{\rho_L^2}\right]^{1/4}(1 - \alpha_{SB})^{3/2} \tag{53}$$

7. Rise velocity of small bubbles in liquid slug region

$$U_{GLS} = U_{LLS} + 1.41\left[\frac{g\sigma(\rho_L - \rho_G)}{\rho_L^2}\right]^{1/4}(1 - e_{LS})^{3/2} \tag{54}$$

8. Average translation velocity of bubble-slug unit

$$U_{BSU} = 1.25\, U_M + 1.41\left[\frac{g\sigma(\rho_L - \rho_G)}{\rho_L^2}\right]^{1/4} \tag{57}$$

9. Average liquid velocity in large bubble region

$$U_{LLB} = \frac{U_{max}}{1 - (3/2\, e_{LB})}\Big((1 - 3/2\, e_{LB}) - \tfrac{1}{2}\,(4 - \frac{6U_{SL}}{U_{max}})(1 - (3/2\, e_{LB})^2) + \tfrac{1}{3}\,(3 - \frac{6U_{SL}}{U_{max}})(1 - (3/2\, e_{LB})^3)\Big) \tag{46}$$

10. Approximate liquid center-line velocity

$$U_{max} = U_{BSU} - U_S' \tag{47}$$

11. Average slip velocity between phases

$$U_S' = \frac{U_{SG}}{e_{BSU}} - \frac{U_{SL}}{1 - e_{BSU}} \tag{48}$$

12. Relative void fraction of small bubbles in large bubble region

$$e_{SB} = e_{LS}\,(1 - e_{LB}) \tag{52}$$

13. Approximate total average void fraction of unit

$$e_{BSU} = (1 - e_{LS})\,\beta\, e_{LB} + e_{LS} \tag{58}$$

Table 2

BUBBLE-SLUG MODEL
Solution Algorithm

For any given flow rate pair of superficial velocities in a column of diameter, D, proceed as follows:

1.) It has been reported that the gas void fraction in the liquid slug region, $_{LS}$, is approximately 0.20.

2.) Calculate the average velocity of the bubble-slug unit, U_{BSU}, from equation (57).

3.) Assume a value for $_{LB}$ within the physical limits of 0.00 to 1.0.

4.) Calculate the void fraction of the small bubbles in the large bubble region, $_{SB}$, by equation (52).

5.) Approximate the average total void fraction of the bubble-slug unit, $_{BSU}$, by equation (58).

6.) Approximate the average slip velocity, U_S', by equation (48).

7.) Estimate the center line liquid velocity, U_{max}, by equation (47).

8.) Calculate the average liquid velocity in the large bubble region, U_{LLB}, by equation (46).

9.) Calculate the average liquid velocity in the liquid slug region, U_{LLS}, by equation (26).

10. Calculate the relative large bubble to bubble-slug unit length fraction, , by equation (24).

11.) Calculate the average total void fraction of the bubble-slug unit, $_{BSU}$, by equation (29).

12.) Calculate the average rise velocity of the small bubbles in the large bubble region, U_{GSB}, by equation (53).

13.) Calculate the average rise velocity of the small bubbles in the liquid slug region, U_{GLS}, by equation (54).

14.) Calculate the average rise velocity of the large bubble, U_{GLB}, by equation (25).

15.) Calculate the void fraction of the large bubble, $_{LB}$, by equation (18).

16.) Check assumed value of $_{LB}$ versus the calculated value, iterate, repeating steps 3 through 15, until convergence.

Figure 1.

An Idealized Bubble-Slug Unit

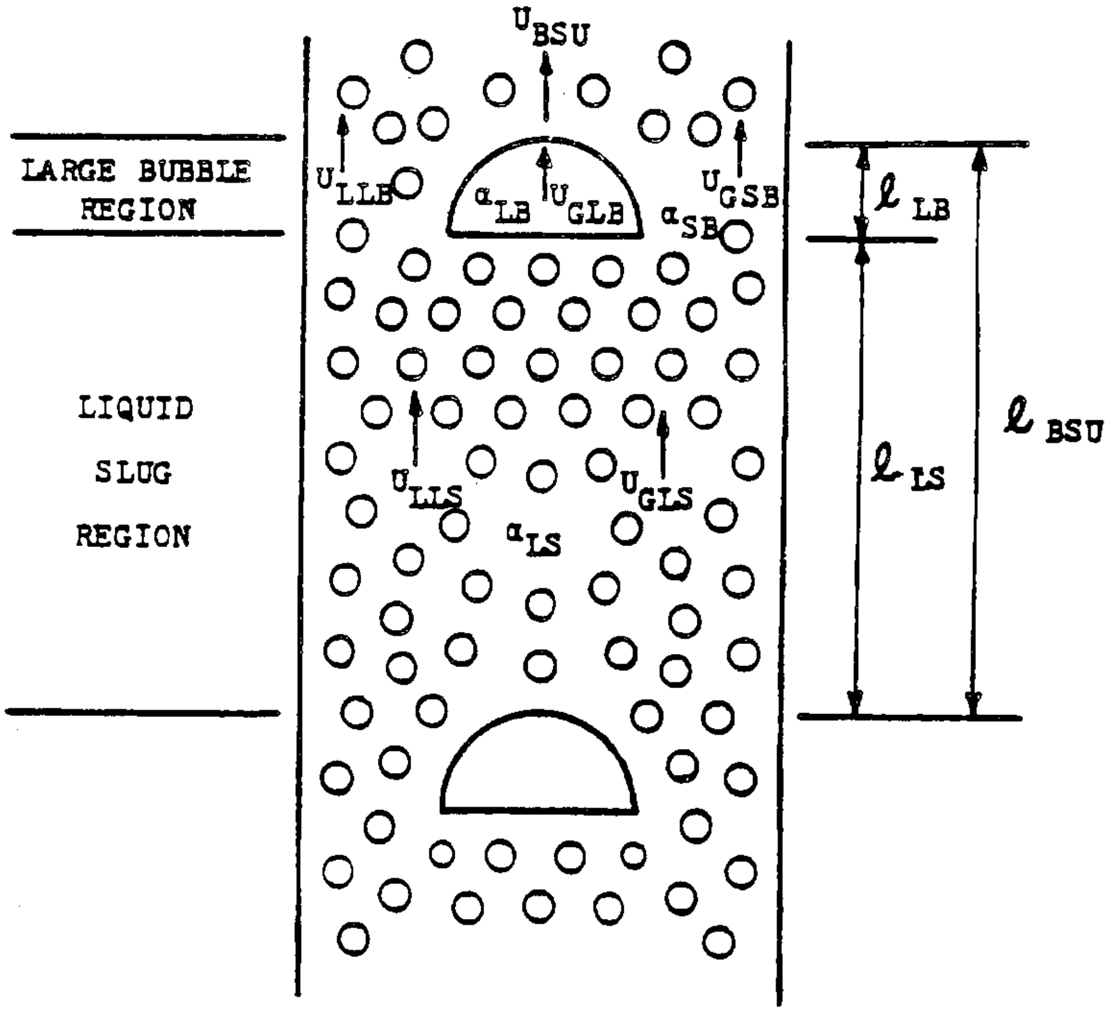

Figure 2.

Comparison of average liquid velocity profiles

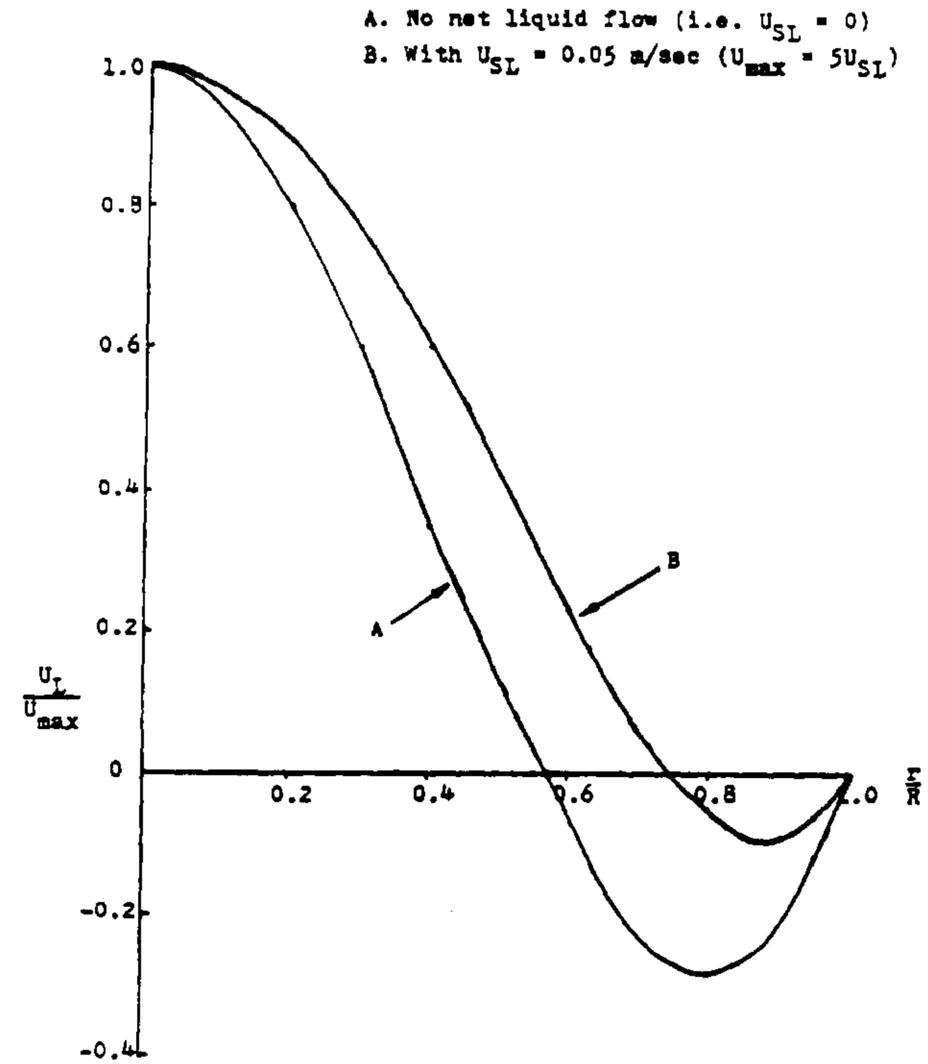

Figure 3 .

Relative Contribution to Total Gas Holdup.
(Reproduced from Beinhauer, 1971)

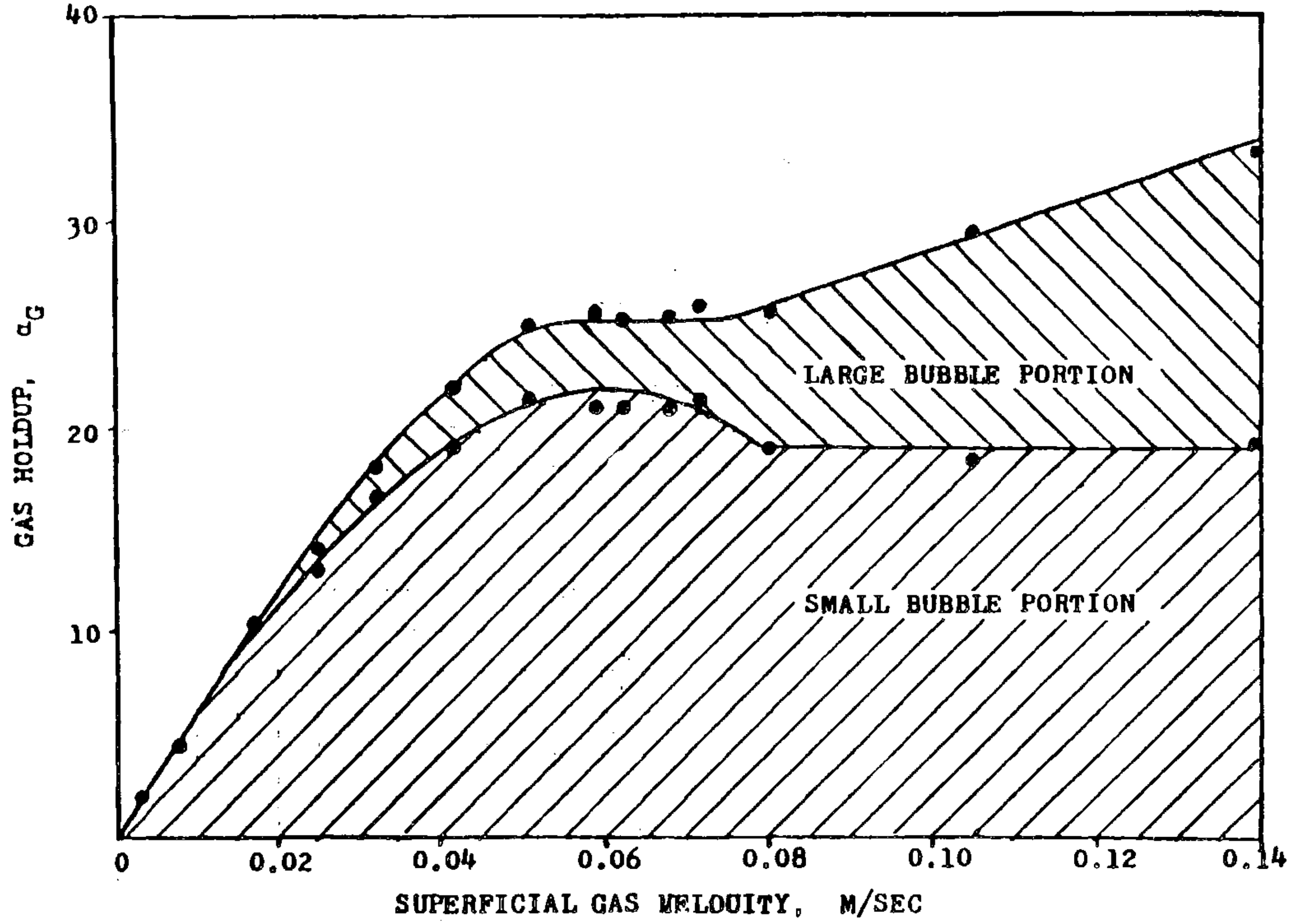

Figure 4.

VERIFICATION FOR UNIQUENESS OF SOLUTION FOR BUBBLE SLUG MODEL