

## CHAPTER 4

### HYDRAULIC MODELING OF GTL TRANSPORTATION

Transportation of GTL products through the TAPS can be achieved by using one of the two possible modes. In the first mode, alternate batches or slugs of crude oil and GTL can be transported through the pipeline. This mode is referred to as batching or slugging. A minimum slug length will be required because some mixing between the crude oil and GTL will take place at the leading and trailing edges of the slugs. In the second mode, the GTL products can be mixed with the crude oil and sent through the pipeline as a single liquid phase. This mode is termed blending or commingling.

In order to study the feasibility of GTL transportation through TAPS, it is necessary to predict the pressure drop or gradient along the entire pipeline. Pressure drops occur due to various forces such as gravitational, frictional, etc. The objective of this study is to solve the pertinent energy equations for both batch and commingled flow modes, and to analytically determine the pressure gradients and related hydraulic flow parameters for each transportation mode. The factors that contribute to this pressure drop are examined, and the methods of accounting for them are considered. A comparison of the pressure gradient calculations is presented for the batching and the commingled flow modes.

#### 4.1 BATCH FLOW

The transportation of GTL and Crude Oil in slugs or batches results in the creation of an interface zone between both fluids. This is analogous to two-phase slug flow in pipelines, in that each batch or slug is followed by an air pocket. This interface zone is made up of mostly air pockets, and a mixture of both fluids. The magnitude of the interface zone is a function of the fluid velocity, density differences, viscosity, pipe diameter, length, time and composition (Baum et al., 1998).

Two-phase flow is a more complex phenomenon than single-phase flow, primarily because the distribution of the two phases is unknown and difficult to specify quantitatively. When gas and liquid flow simultaneously in a pipe, the two phases can distribute themselves in a variety of flow configurations, depending on operating parameters, physical properties of the two-phases, as well as geometrical variables (for purposes of this work, any mention or reference to "gas", is in actuality, a reference to the air pockets between slugs). In addition, the flow is affected by various factors such as the liquid hold-up, void fraction, pressure loss etc.

The fundamental flow patterns in two-phase flow as classified by Baker (1954) are:

- i) Stratified flow: Flow in which the liquid flows along the bottom of the pipe and the gas flows above, over a smooth liquid interface.
- ii) Wavy flow: This is similar to stratified flow except that the gas moves at a higher velocity and the interface is disturbed by waves traveling in the direction of flow.
- iii) Slug flow: Flow in which a wave is picked up periodically by the more rapidly moving gas, to form a frothy slug which passes through the pipe at a much greater velocity than the average liquid velocity.

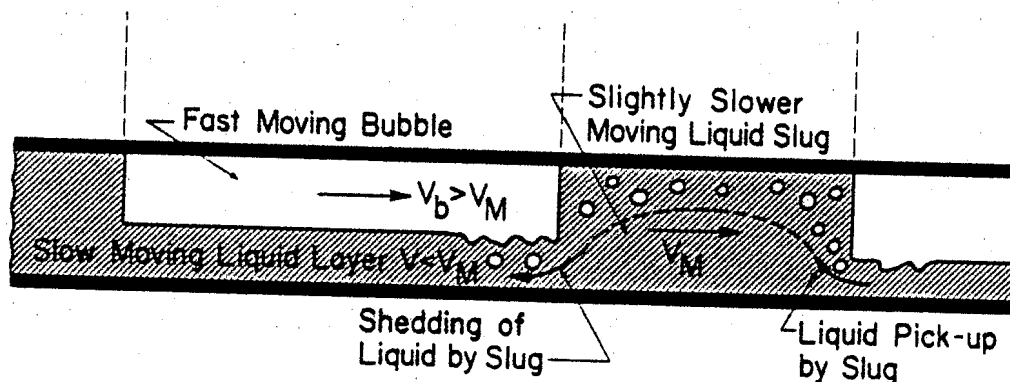
- iv) Plug flow: Flow in which alternate plugs of liquid and gas move along the upper part of the pipe.
- v) Bubble flow: Flow in which bubbles of gas move along the upper part of the pipe at approximately the same velocity as the liquid.
- vi) Annular flow: Flow in which the liquid forms a film around the inside wall of the pipe and the gas flows at a high velocity as a central core.
- vii) Spray flow: Flow in which most or nearly all of the liquid is entrained as a spray by the gas.

These flow patterns have been further classified into four major types: Stratified Flow (Stratified Smooth and Stratified Wavy), Intermittent Flow (Elongated Bubble Flow and Slug Flow), Annular Flow (Annular Mist Flow and Annular Wavy Flow), and Dispersed Flow (Taitel et al, 1976; Aziz et al, 1978).

Slug flow occurs because of the velocity difference in the flow of gas and liquids. The liquid phase grows in amplitude until; it succeeds in bridging the entire cross-section of the pipe to form a "slug". The slug is immediately accelerated to an average stable velocity, by the gas behind it (Govier and Aziz, 1972). The length of the gas bubble depends on the flow rates and the fluid properties, and for given flow rates, it depends on the manner in which the fluids are introduced. It also depends on the system pressure and therefore increases as the pressure declines in the direction of flow (Govier and Aziz, 1972).

Various models have been proposed to account for or describe slug flow in horizontal pipes or tubes. Kordyban (1961) was the first to propose such a model. In his model, the liquid slug moves at the average velocity of the gas bubble and "skates" over the top of the more slowly moving liquid below it. Based upon this concept, a pressure drop expression was developed. Govier and Aziz (1972) later discovered that the model was oversimplified and inadequate.

Dukler and Hubbard (1975) presented a model that until today remains the reference point for the analysis of gas-liquid slug flow in pipes (Figure 4.1). The model permits the prediction in detail of the unsteady hydrodynamic behavior of gas-liquid slug flow. It is based on the observation that a fast moving slug overruns a slow moving liquid film, accelerating it to full slug velocity in a mixing eddy located at the front of the slug. A new film is shed behind the slug ("scooping mechanism") that decelerates with time.



**FIGURE 4.1** Schematic Representation of the Dukler and Hubbard Model

The model is based on the following assumptions:

- i) Steady state representation of the slug.
- ii) Mixing in the slug is a result of a mixing eddy and diffusion due to turbulence.
- iii) Slug length is constant.
- iv) Amount of liquid scooped at the head of the liquid is equal to the amount of liquid shed at its tail.
- v) Pressure drop across the film is negligible.

The model has the ability to predict the slug fluid velocity, length of the slug, film region behind the slug, film distance as a function of time and distance, as well as the pressure drop (containing an acceleration and frictional term) across the slug. In 1989, Kokal et al. highlighted a shortcoming of the model, in that it requires the values of slug frequency and liquid hold-up in the liquid slug, which are difficult to estimate.

Over the years, various workers have modified the basic assumptions inherent in the Dukler and Hubbard model, and have derived new models or procedures for obtaining the parameters required for the description of slug flow. A review of these models is available in the literature (Akwukwaegbu, 2001).

## 4.2 COMMINGLED FLOW

In this mode of transportation, the Crude Oil and GTL are blended, before being sent through the pipeline, as a single liquid phase mixture. This mode is termed blending or commingling. The transport of such fluid mixtures in horizontal or nearly horizontal pipes has become the norm, especially in the gathering and processing of hydrocarbons. This enables major cost savings in pipeline construction, and permits the centralization of processing facilities. This usually results in the improvement of processing economics and conservation of resources.

When a mixture of fluids flows in a system, the component fluids can be distributed in a variety of flow configurations or patterns, depending on the operating parameters, physical properties of the fluids, as well as geometrical variables. The flow may also be affected by pressure losses in the system, liquid holdup (as a result of density differences) etc.

Since GTL and Crude Oil are both hydrocarbons, and as such may have very similar fluid properties, the possibility exists of blending both fluids into one homogeneous mix. This is subject to laboratory testing to determine the actual fluid properties of the resulting fluid mixture.

As part of the GTL project, tests were conducted by the Petroleum Engineering Department at the University of Alaska Fairbanks, on samples of GTL and Crude Oil (Ramakrishnan, 2000). From the results of the tests, it was observed that when both fluids were mixed, they blended into a single homogeneous liquid. There was no separation into distinct layers or boundaries when the mixture was left to stand.

This then allows the flexibility of treating the mixture as a single-phase homogeneous liquid, with its own unique fluid properties. In studying the commingled flow of GTL and Crude Oil through the Trans-Alaska Pipeline System, the Bernoulli equation of pressure for the flow of fluids in pipes is used. This equation forms the basis for any analysis in the area of fluid mechanics, and has been discussed in detail, by a great number of researchers.

### 4.3 DEVELOPMENT OF MODEL EQUATIONS

In studying the flow of Gas To Liquids and Crude Oil through the Trans Alaska Pipeline System (TAPS), in either batch or commingled mode, the primary concern will be on the expected pressure drop or gradient along the entire pipeline. Such pressure drop may be due to a number of reasons, such as friction, hydrostatics etc. In carrying out a proper study, the various factors that contribute to this pressure drop are examined, and the methods of accounting for them are considered. This will be achieved by presenting mathematical models or equations, which are used to obtain numerical values for these factors, and as such, allow a proper understanding of the role played by these factors in the hydraulics.

#### 4.3.1 Batch Flow Model

In this transport mode, alternate batches or slugs of crude oil and GTL can be transported through the pipeline. This mode is also referred to as batching or slugging. A minimum slug length will be required because some mixing between the crude oil and GTL will take place at the leading and trailing edges of the slugs. The study of the expected pressure drop, that occurs during transportation in slugs or batches will focus on the minimum slug length, length of the interface (or void space) between the slugs, as well as the length of the mixing zone. Development of batch flow model equations is described in the following sections.

##### 4.3.1.1 Assumptions

In studying the batching or slugging mode of transport, the following assumptions have been made:

- i) Incompressible fluid flow, steady state and fully developed.
- ii) Constant slug length.
- iii) The bubble (void) between the slugs is occupied by air.
- iv) The liquid film has a constant thickness.
- v) Flow is isothermal with constant fluid properties
- vi) There is some degree of mixing between the trailing film edge and the head of the slug.

##### 4.3.1.2 Governing Equations

The slug body is divided into two sections (see Figure 4.2), the liquid slug zone of length  $l_s$ , and the mixing zone of length,  $l_m$ . In the original work, the mixing zone was construed to consist of a liquid film, and an elongated air bubble (Taitel, et al, 1990). For this work, this definition has been modified, such that the mixing zone is the interface between slugs.

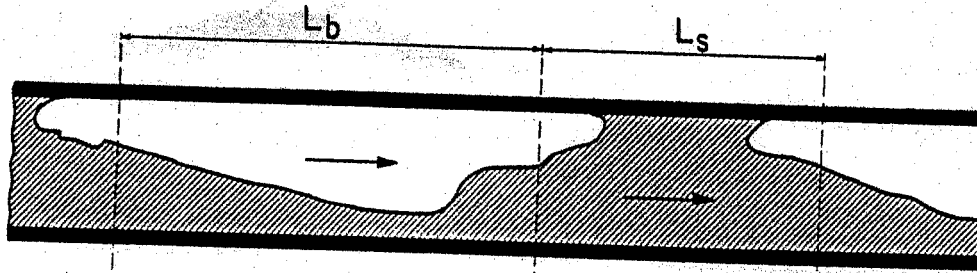


FIGURE 4.2 Schematic Representation of Slug Flow (Govier and Aziz, 1972)

The pressure drop across one slug unit is calculated from

$$\Delta P = \Delta P_f + \Delta P_a + \Delta P_h \quad (4.1.1)$$

where  $\Delta P_f, \Delta P_a, \Delta P_h$  are the pressure drops due to friction, acceleration, and hydrostatic forces respectively (Kokal, et al., 1989; Taitel, et al, 1990). The pressure drops are affected by the flow regime of the fluid i.e. laminar (streamlined) or turbulent.

#### 4.3.1.2.1 Pressure Drop Due To Friction

This is the pressure drop due to frictional forces within the liquid slug and the void (air pocket and liquid film). Taitel and Barnea (1990) presented Equation (4.1.2) in order to determine the pressure drop due to friction. It is a combination of the friction forces produced by the individual components of a typical slug.

$$\Delta P_f = \frac{2f_s \rho_m V_m^2 l_s}{D} + \frac{2f_g \rho_g V_g^2 l_g}{D_g} + \frac{2f_f \rho_l V_f^2 l_f}{D_f} \quad (4.1.2)$$

where the friction factors of the slug,  $f_s$ , air bubble,  $f_g$ , and liquid film (fluid interface zone),  $f_f$  are based on the Reynolds number of the slug,  $R_{es}$ , air bubble,  $R_{eg}$ , and the film,  $R_{ef}$ . For this work, it is assumed that the effects of the air pocket or bubble, are negligible, hence Equation (4.1.2) then becomes;

$$\Delta P_f = \frac{2f_s \rho_l V_m^2 l_s}{D} + \frac{2f_f \rho_m V_f^2 l_m}{D_f} \quad (4.1.3)$$

The Moody friction factor is applied for laminar flow regime, and is defined as:

$$f = \frac{64}{R_e} \quad (4.1.4)$$

The Zigrang and Sylvester (1985) equation for turbulent flow, which incorporates the pipe roughness factor,  $\epsilon$ , can be given by:

$$\frac{1}{\sqrt{f}} = -2 \log \left[ \frac{\epsilon/D}{3.7} - \frac{5.02}{N_{Re}} \log \left( \frac{\epsilon/D}{3.7} + \frac{13}{N_{Re}} \right) \right] \quad (4.1.5)$$

The Reynolds number for the slug, and film respectively, are obtained from the following expressions:

$$R_{es} = \frac{DV_m \rho_l}{\mu_l} \quad (4.1.6)$$

$$R_{emz} = \frac{D_f V_f \rho_{mz}}{\mu_{mz}} \quad (4.1.7)$$

Where,

$$\rho_{mz} = \rho_{l1} E_{ls} + (1 - E_{ls}) \rho_{l2} \quad (4.1.8)$$

$$\mu_{mz} = \mu_{l1} E_{ls} + (1 - E_{ls}) \mu_{l2} \quad (4.1.9)$$

$\rho_{mz}$ ,  $\rho_{l1}$  and  $\rho_{l2}$ , are the densities of the mixing zone and slugs respectively;  $\mu_{mz}$ ,  $\mu_{l1}$ , and  $\mu_{l2}$  are the viscosities of the mixing zone and slugs respectively;  $E_{ls}$ , is the liquid holdup in the liquid slug;  $E_{lf}$ , is the liquid holdup in the interface zone;  $D_f$  is the hydraulic diameter occupied by the interface zone.

#### 4.3.1.2.2 Pressure Drop Due To Acceleration

The film velocity,  $V_f$ , just before slug pick-up, is lower than the velocity in the main body of the slug,  $V_s$ . This necessitates the acceleration of the film to match the velocity of the slug. As a result, there is a pressure drop generated by this, and it can be defined as (Kokal et al, 1989):

$$\Delta P_a = \rho_l E_{ls} (V_s - V_f) (V_s - V_f) \quad (4.1.10)$$

#### 4.3.1.2.3 Hydrostatic Pressure Drop

This pressure drop can be experienced in any system because of the pipe orientation or inclination. Equation (4.1.11) was presented by Kokal et al (1989) and Taitel et al (1990) to determine the pressure drop due to pipe inclination.

$$\Delta P_h = \rho_{ms} (g \sin \beta) l_s + \rho_f (g \sin \beta) l_f \quad (4.1.11)$$

where:

$$\rho_f = \rho_{l1} E_{lf} + (1 - E_{lf}) \rho_{l2} \quad (4.1.12)$$

$\beta$  is the angle of inclination. Since  $\sin \beta = h/L = \Delta z/L$ , equation (4.1.11) can be re-written as

$$\Delta P_h = (\rho_{ms} l_s + \rho_f l_f) g \Delta z/L \quad (4.1.13)$$

For the purposes of this work, the Equation (4.1.13) is presented as

$$\Delta P_h = (\rho_{l1} l_s + \rho_{ms} l_m) g \Delta z/L \quad (4.1.14)$$

The schematic arrangement of the batches or slugs, is as shown in Figure 4.3.

The total pressure drop across the slug can be calculated from the sum of equations (4.1.3), (4.1.10) and (4.1.14). This would require the determination of the following quantities: slug length,  $l_s$ ; liquid hold-up in the slug,  $E_{ls}$ ; average fluid velocity in the slug,  $V_s$ ; film velocity,  $V_f$ ; and length of the mixing zone,  $l_m$ .

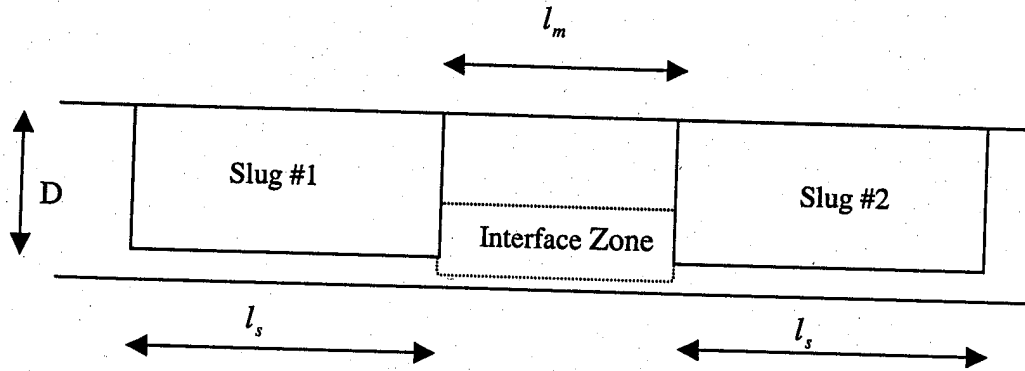


FIGURE 4.3 Schematic Representation of Batch or Slug Flow

#### 4.3.1.2.4 Slug Length

This is the length of a slug. In 1986, Scott et al. presented a correlation for the determination of the slug length for large diameter pipes, and which is given by:

$$\ln(l_s) = -25.4144 + 28.4948(\ln(D))^{0.1} \quad (4.1.15)$$

#### 4.3.1.2.5 Average Fluid Velocity

By conducting a momentum balance over a slug unit, the average fluid velocity is given by (Govier et al, 1972; Kokal et al, 1989; Taitel et al, 1990; Fan et al, 1993; Sharma et al, 1998).

$$V_m = \frac{Q}{A} = \frac{Q_1 + Q_2}{A} = V_{s1} + V_{s2} \quad (4.1.16)$$

where  $V_{s1}$  and  $V_{s2}$  are the superficial velocities of the slugs respectively. The average slug velocity,  $V_s$ , can be determined from equation (4.1.16) by setting it equal to the average fluid velocity.

$$V_m = V_s \quad (4.1.17)$$

#### 4.3.1.2.6 Transitional Velocity

This is the slug transitional velocity. This can also be defined as the velocity of the leading edge of the slug. In 1990, Taitel and Barnea, presented a correlation, which is actually a linear combination of the interface velocity.

$$V_t = C_o V_s + V_d \quad (4.1.18)$$

where  $C_o = 2$  for laminar flow,  $C_o = 1.2$  for turbulent flow, and  $V_d$  is the propagation or drift velocity and is defined as (Kokal et al, 1989):

$$V_d = 0.345 \sqrt{\frac{gD(\rho_{l1} - \rho_{l2})}{\rho_{l1}}} \quad (4.1.19)$$

#### 4.3.1.2.7 Liquid Slug Hold-up

When there is a difference in phase properties (density and/or viscosity), one of them, usually the less dense phase, tends to flow at a higher in situ average velocity than does the other. This gives rise to the existence of slip of one phase past the other, or holdup of one phase relative to the other. In 1996, Abdul-Majeed presented a correlation for the determination of the liquid holdup in the slug. It is a modification of the Lockhart-Martinelli parameter (1949). Equations (4.1.20) and (4.1.21) are for turbulent and laminar flow regimes respectively.

$$(E_{ls})_{theoretical} = \exp(-0.9304919 + 0.5285852R - 9.219634 \times 10^{-2} R^2 + 9.02418 \times 10^{-4} R^4) \quad (4.1.20)$$

$$(E_{ls})_{theoretical} = \exp(-1.099924 + 0.6788495 R - 0.1232191 \times 10^{-2} R^2 - 1.778653 \times 10^{-3} R^3 + 1.626819 \times 10^{-3} R^4) \quad (4.1.21)$$

where  $R = \ln(X)$ , and



$$X = \left[ \frac{V_{s2} \rho_{l2} \mu_{l1}}{V_{s1} \rho_{l1} \mu_{l2}} \right]^m \frac{\rho_{l1} V_{s1}^2}{\rho_{l2} V_{s2}^2} \quad (4.1.22)$$

X is the Lockhart-Martinelli parameter (1949), and  $m = 0.2$  for turbulent flow and  $m = 1$  for laminar flow. Due to assumptions made in the development of the model, a correction was made to the value of the liquid holdup obtained from both equations:

$$(E_{ls})_{actual} = C(E_{ls})_{theoretical} \quad (4.1.23)$$

where

$$C = 0.528(V_{s2} V_{s1})^{-0.215121} \quad (4.1.24)$$

#### 4.3.1.2.8 Interface Velocity

From the original data of Dukler and Hubbard (1975) model, the film velocity is given as

$$V_f = V_m \left( \frac{1}{1 + \frac{0.2V_m}{\omega}} \right) \quad (4.1.25)$$

where  $\omega$  is the slug frequency, and is given by equation (4.1.26) as (Govier et al, 1972)

$$\omega = 0.0226 \left[ \frac{V_{sl}}{gD} \left( \frac{19.75}{V_m} + V_m \right) \right]^{1.2} \quad (4.1.26)$$

#### 4.3.1.2.9 Length of the Mixing Zone

This is the interface region between slugs. This interface zone is made up of mostly air pockets, and a mixture of both fluids. The magnitude of the interface zone is a function of the fluid velocity, density differences, viscosity, composition, time, pipe diameter and length. It is characterized by a rapidly varying liquid hold-up. This was originally presented in the Dukler and Hubbard model (1975) as,

$$l_m = \frac{0.15}{g} (V_m - V_f)^2 \quad (4.1.27)$$

It is observed that at large values of  $V_m$ , equation (4.1.27) largely over predicts  $l_m$ . In 1993, Andreussi et al. proposed a new correlation that corrects such over predictions, and is given by:

$$l_m = k_m (1 - E_{ls}) D \quad (4.1.28)$$

where  $k_m$  is a factor for the length of the mixing zone and is approximately equal to 30.

#### 4.3.1.2.10 Liquid Hold-up in the Mixing Zone

At steady state, the mass exchange rate between the liquid slug and the film is expressed as (Govier et al., 1972; Dukler and Hubbard, 1975; Nicholson et al., 1978; Kokal et al., 1989; Taitel et al., 1990):

$$\rho_l A E_{ls} (V_t - V_s) = \rho_l A E_{lf} (V_t - V_f) = n k_d^R \quad (4.1.29)$$

From equation (4.1.28), the film hold-up can be obtained as

$$E_{lf} = E_{ls} \frac{(V_t - V_s)}{(V_t - V_f)} \quad (4.1.30)$$

#### 4.3.1.2.11 Interface Hydraulic Diameter

This is fraction of the actual pipe diameter occupied by the film (interface). In calculating the hydraulic diameter, the approach presented by Darby (1996) will be followed. If the height of the interface within the pipe is given as  $h$  (which can either be smaller or larger than the radius of the pipe,  $R$ ), then the cross-sectional area can be obtained from equation (4.1.31a)

$$A = R^2 \left[ \cos^{-1} \left( 1 - \frac{h}{R} \right) - \left( 1 - \frac{h}{R} \right) \sqrt{1 - \left( 1 - \frac{h}{R} \right)^2} \right] \quad (4.1.31a)$$

From equation (4.1.31b), the wetted perimeter can be calculated as:

$$W_p = 2R \cos^{-1} \left( 1 - \frac{h}{R} \right) \quad (4.1.31b)$$

As a result, the interface hydraulic diameter can then be calculated from:

$$D_f = \frac{4A E_{lf}}{W_p} \quad (4.1.31c)$$

Setting the change in elevation equal to the head loss due to friction initializes this iterative procedure,

$$\Delta z = h_f = \frac{2f_f L Q^2}{g D_f A^2} \quad (4.1.31d)$$

which is outlined as follows:

- i) A value is assumed for  $h/R$ , and the parameters  $A$ ,  $W_p$  and  $D_f$  are determined from equations (4.1.31a), (4.1.31b) and (4.1.31c) respectively.
- ii) From equation (4.1.7) the interface Reynolds number,  $N_{R_{if}}$ , is calculated.
- iii) The interface frictional factor,  $f_f$ , can be computed as function of  $N_{R_{if}}$  by using equations (4.1.4) and (4.1.5).
- iv) By assuming values for  $h/R$ , an iterative procedure is applied to obtain solutions to the right hand side (RHS) of equation (4.1.31d). The guessed values of  $h/R$  are continuously adjusted until a tolerance limit is reached.

#### 4.3.1.2.12 Average Pressure Gradient

The average pressure gradient is determined for one complete slug unit, by dividing the total pressure drop across a slug, by the effective slug length. This is given by equation (4.1.32) as

$$\frac{\Delta P}{L} = \frac{\Delta P_f + \Delta P_a + \Delta P_h}{l_s} \quad (4.1.32)$$

### 4.3.2 Commingled Flow Model

In this transport mode, the GTL and Crude Oil are pre-mixed before shipment through the TAPS as a single phase. For the purpose of this analysis, it is assumed that the fluids are homogeneously mixed, and that due to the envisioned throughput, there will be no separation into distinct layers.

#### 4.3.2.1 Assumptions

In studying the commingled mode of transport, the following assumptions will have to be made:

- i) Incompressible fluid flow, steady state and fully developed
- ii) Flow is isothermal with constant fluid properties.
- iii) Fluid exhibits Newtonian behavior
- iv) No separation into constituent fluids.

#### 4.3.2.2 Governing Equations

Consider a finite element of an inviscid (frictionless) fluid, subject only to the action of gravity, (i.e. the fluid is at rest). Applying Newton's third law of motion to this fluid element (Landau et al., 1959; Bird et al., 1960; Kaufmann, 1963; Streeter et al., 1985)

$$F_s = dm \frac{dv}{dt} \quad (4.2.1)$$

where  $F_s$ , is the resultant of all external forces in the direction of the streamline;  $v$ , is the fluid velocity; and,  $dm$ , is the mass of the element.

The forces acting on the element are the weight and the end forces (pressure difference between the upper and lower faces), as shown in Figure 4.4. Thus, from equation (4.2.1),

$$\rho g \cdot ds \cdot dA \cdot \cos \theta + P \cdot dA - dA \left( P + \frac{\partial P}{\partial s} ds \right) = \rho \cdot ds \cdot dA \frac{\partial v}{\partial t} \quad (4.2.2)$$

Since  $\theta$  is the angle sustained by the particle with the horizontal,

$$\cos \theta = -\frac{\partial z}{\partial s} \quad (4.2.3)$$

Equation (4.2.2) then becomes:

$$-\left( \rho g \cdot ds \cdot dA \frac{\partial z}{\partial s} \right) - \left( ds \cdot dA \cdot \frac{\partial P}{\partial s} \right) = \rho \cdot ds \cdot dA \frac{dv}{dt} \quad (4.2.4)$$

Equation (4.2.4) then simplifies to:

$$\frac{dv}{dt} = -g \frac{\partial z}{\partial s} - \frac{1}{\rho} \frac{\partial P}{\partial s} \quad (4.2.5)$$

In general, the fluid velocity,  $v$ , is a function of both time and location,  $s$ , along the streamline. Therefore, the total derivative for the velocity term is given as,

$$dv = \frac{\partial v}{\partial s} ds + \frac{\partial v}{\partial t} dt \quad (4.2.6)$$

Since, velocity is the rate of change of distance with time, the actual acceleration of the particle in the direction of flow becomes:

$$\frac{dv}{dt} = v \frac{\partial v}{\partial s} + \frac{\partial v}{\partial t} \quad (4.2.7)$$

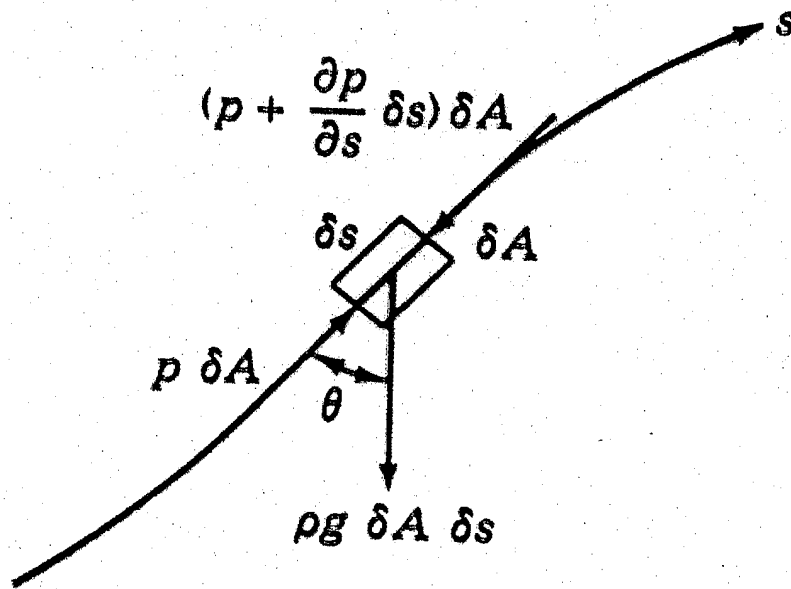


FIGURE 4.4 Force Balance on a Finite Element (Streeter, 1985)

On the assumption of steady state flow, equation (4.2.7) reduces to,

$$\frac{dv}{dt} = v \frac{\partial v}{\partial s} \quad (4.2.8)$$

This on substitution into equation (4.2.5) and re-arranging yields

$$v \frac{\partial v}{\partial s} + g \frac{\partial z}{\partial s} + \frac{1}{\rho} \frac{\partial P}{\partial s} = 0 \quad (4.2.9)$$

Since the distance,  $s$ , is the only independent variable, the partial derivatives are replaced by total derivatives, and thus, equation (4.2.9) then becomes

$$\frac{1}{\rho} \frac{dP}{ds} + g \frac{dz}{ds} + v \frac{dv}{ds} = 0 \quad (4.2.10)$$

Equation (4.2.10) is best known as the Euler's equation of motion along a streamline (Landau et al, 1959).

All the terms in equation (4.2.10) are derivatives with respect to distance,  $s$ . This then enables the integration along the streamline to obtain

$$\frac{v^2}{2} + \frac{P}{\rho} + gz = C_e \quad (4.2.11)$$

where  $C_e$  is a constant. Equation (4.2.11) is more commonly known as the Bernoulli equation of pressure in steady flow or the equation of energy for steady flow.

For flow between points 1 and 2, equation (4.2.11) is written as,

$$\left( \frac{V_2^2}{2g} + \frac{P_2}{\rho_2 g} + z_2 \right) - \left( \frac{V_1^2}{2g} + \frac{P_1}{\rho_1 g} + z_1 \right) = C_e \quad (4.2.12)$$

In equation (4.2.12),  $V^2/2g$ , and  $P/(\rho g)$  are the velocity and pressure heads respectively. The last term,  $z$ , is the elevation or geometric head of the fluid above an arbitrary reference plane (Kaufmann, 1963; Holland, 1973).

#### 4.3.2.2.1 Energy Losses

Since most natural liquids are very nearly incompressible (i.e. constant density), they are not inviscid (frictionless). Internal friction (viscosity) converts part of the flow energy into other energy forms such as sound, heat etc. and it is "lost" (Kaufmann, 1963). This loss is normally considered as a "head", the friction head,  $h_f$ , and is given by the Darcy-Weisbach equation (Smith et al, 1960) as:

$$h_f = 4f \frac{LV^2}{2gD} \quad (4.2.13)$$

Therefore, equation (4.2.12) is re-written as,

$$\left( \frac{V_1^2}{2g} + \frac{P_1}{\rho g} + z_1 \right) = \left( \frac{V_2^2}{2g} + \frac{P_2}{\rho g} + z_2 \right) + h_f \quad (4.2.14)$$

For steady incompressible flow through a pipe, between points 1 and 2, with a pump at one end, equation (4.2.14) can be re-written as,

$$\left( \frac{V_2^2}{2g} + \frac{P_2}{\rho g} + z_2 \right) - \left( \frac{V_1^2}{2g} + \frac{P_1}{\rho g} + z_1 \right) = \Delta h_p - h_f \quad (4.2.15)$$

where  $\Delta h_p$ , is the head imparted to the fluid by the pump (Holland, 1973).

This then implies that the total pressure drop across the streamline is given as

$$\frac{P_1 - P_2}{\rho g} = \frac{(V_2^2 - V_1^2)}{2g} + (z_2 - z_1) + (h_f - \Delta h_p) \quad (4.2.16)$$

or simply

$$\Delta P = \rho g \left[ \left( \frac{V_2^2 - V_1^2}{2g} \right) + (z_2 - z_1) + (h_f - \Delta h_p) \right] \quad (4.2.17)$$

Equation (4.2.17) will form the basis for the study of the commingled flow of GTL and Crude Oil through TAPS.

#### 4.4 APPLICATION OF MODEL EQUATIONS

In choosing the appropriate mode for transporting GTL through TAPS, i.e. either batch or commingled flow, the derived model equations will have to be applied to estimate the expected pressure drop for each mode. Based on the results obtained from the computations, a reasonable choice can then be made.

##### 4.4.1 Calculation Algorithm

The Trans-Alaska Pipeline System (TAPS) is an 800 miles long and 48" diameter pipeline. For computational purposes, it has been divided into six (6) major pipe sections. These sections are as follows:

- i. Pump Station #1 to Pump Station #3 (Length,  $L = 104.27$  mi.; Change in elevation,  $\Delta z = +1344.3$  ft.)
- ii. Pump Station #3 to Pump Station #4 ( $L = 39.79$  mi.;  $\Delta z = +1380$  ft.)
- iii. Pump Station #4 to Pump Station #7 ( $L = 270.02$  mi.;  $\Delta z = -1859.1$  ft.)
- iv. Pump Station #7 to Pump Station #9 ( $L = 134.66$  mi.;  $\Delta z = +604.3$  ft.)
- v. Pump Station #9 to Pump Station #12 ( $L = 186.36$  mi.;  $\Delta z = +312.6$  ft.)
- vi. Pump Station #12 to Valdez Terminal ( $L = 65.1$  mi.;  $\Delta z = -1655.4$  ft.)

The successful application of the model equations requires a prior knowledge of fluid properties, such as density and viscosity. Also important, is the knowledge of the pipe parameters (diameter, length, geometry), as well as current operating conditions (flow rate, pump information, pipe specifications). The systematic procedures necessary for the determination of the total pressure drop, as well as the average pressure gradient, are outlined in the following sections.

##### 4.4.1.1 Batch Flow

For this transport mode, the focus will also be on the determination of the average slug length, length of the mixing zone, and liquid holdup in the slug.

The sequential steps, which are carried out for each pipe section, are outlined as follows:

- i. From equation (4.1.16), the mixture velocity,  $V_m$ , is calculated as a function of the fluid flow rates.
- ii. The transitional velocity,  $V_t$ , is calculated by combining equations (4.1.18) and (4.1.19).
- iii. The determination of the liquid holdup in the slug is a four (4) step process, which can be listed as;
  - a) Determine the Lockhart-Martinelli parameter,  $X$ , from equation (4.1.22)
  - b) From equation (4.1.24), the correction factor,  $C$ , is obtained.
  - c) The theoretical liquid holdup is obtained from either equations (4.1.20) or (4.1.21).
  - d) Using the value obtained for  $C$  from (b) above, the true liquid holdup is calculated using equation (4.1.23).
- iv. The length of the slug,  $l_s$ , is obtained by using equation (4.1.15).
- v. From equation (4.1.28), the length of the mixing zone,  $l_m$ , is calculated.
- vi. The interface velocity,  $V_f$ , is obtained from equation (4.1.25), as a function of  $V_m$ , and the slug frequency,  $\omega$ , obtained from equation (4.1.26).
- vii. From equations (4.1.31a-d), a value for the effective diameter of the interface or film, is obtained.
- viii. Using equations (4.1.6) and (4.1.7), the Reynolds number,  $N_{Re}$ , for the slug, and film, are calculated as functions of densities, velocities, diameters, and viscosities.
- ix. Depending on the flow regime, the appropriate friction factor,  $f$ , is calculated as a function of the Reynolds' number, using either equation (4.1.4) or (4.1.5).
- x. The pressure drop due to friction,  $\Delta P_f$ , is calculated from equation (4.1.2).
- xi. The pressure drop due to acceleration,  $\Delta P_a$ , is calculated from equation (4.1.10).
- xii. The hydrostatic pressure drop,  $\Delta P_h$ , is calculated from equation (4.1.14).
- xiii. The average pressure gradient,  $\Delta P/L$ , is calculated from equation (4.1.32).

Finally, the total pressure drop is computed as the sum of the individual pressure drops across each pipe section.

#### 4.4.1.2 Commingled Flow

In this mode, since there is prior mixing of both GTL and Crude Oil before transport, the analysis will be conducted similar to that of a single-phase fluid. The focus will also be on the expected pressure drop across each pipe segment.

The sequential steps, which are carried out for each pipe section, are outlined as follows:

- i. The initial fluid velocity,  $V_1$ , is calculated as a function of fluid flow rate,  $Q$ , and pipe cross-sectional area,  $A$  (similar to equation (4.1.15)).
- ii. From equation (4.1.5), the Reynolds' number,  $N_{Re}$ , is calculated, in order to determine the appropriate flow regime (for laminar flow,  $N_{Re} \leq 2000$ , and for turbulent flow,  $N_{Re} > 2000$ ).



- iii. Depending on the flow regime, the appropriate friction factor,  $f$ , is calculated as a function of the Reynolds' number, using either equation (4.1.3) or (4.1.4).
- iv. From equation (4.2.13), the head loss due to friction,  $h_f$ , is calculated as a function of the friction factor.
- v. Based on the flow rates and number of pumps in service, the head imparted to the fluid by the pumps,  $\Delta h_p$ , can be determined (Note: Since this analysis is based on already existing equipment, this data would have to be obtained from the pump design and specification sheet).
- vi. The pressure drop,  $\Delta P$ , is determined from equation (4.2.17) (Note: Steady state flow, therefore,  $V_1 = V_2 = V$ ).

The total pressure drop is the sum of the individual pressure drops across each pipe section. In general, the total pressure drop,  $\Delta P_t$ , is calculated as follows:

$$\Delta P_t = \Delta P_1 + \Delta P_2 + \Delta P_3 + \Delta P_4 + \Delta P_5 + \Delta P_6$$

#### 4.5 RESULTS

The calculation path for each mode has been transcribed into computer code for use in the Microsoft Excel® Spreadsheet program. The code is written in the Visual Basic environment. Pressure profiles along the entire length of TAPS for the batch mode and the commingled mode are calculated using the procedures described above. The input data and the results are summarized below.

For batch mode, the pressure gradients in each of the six pipeline sections are calculated for a daily throughput of 1.1MMBPD of both Crude Oil and GTL. Other necessary data are shown below.

Inlet Temperature = 90°F  
 Crude Oil Specific Gravity = 0.8614  
 Crude oil viscosity = 6.2 cp  
 GTL Specific Gravity = 0.73  
 GTL viscosity = 1.0 cp  
 Pipe Diameter = 48 in. = 4 ft  
 Pipe roughness = 0.00001 ft  
 Interface Diameter ratio = 0.3

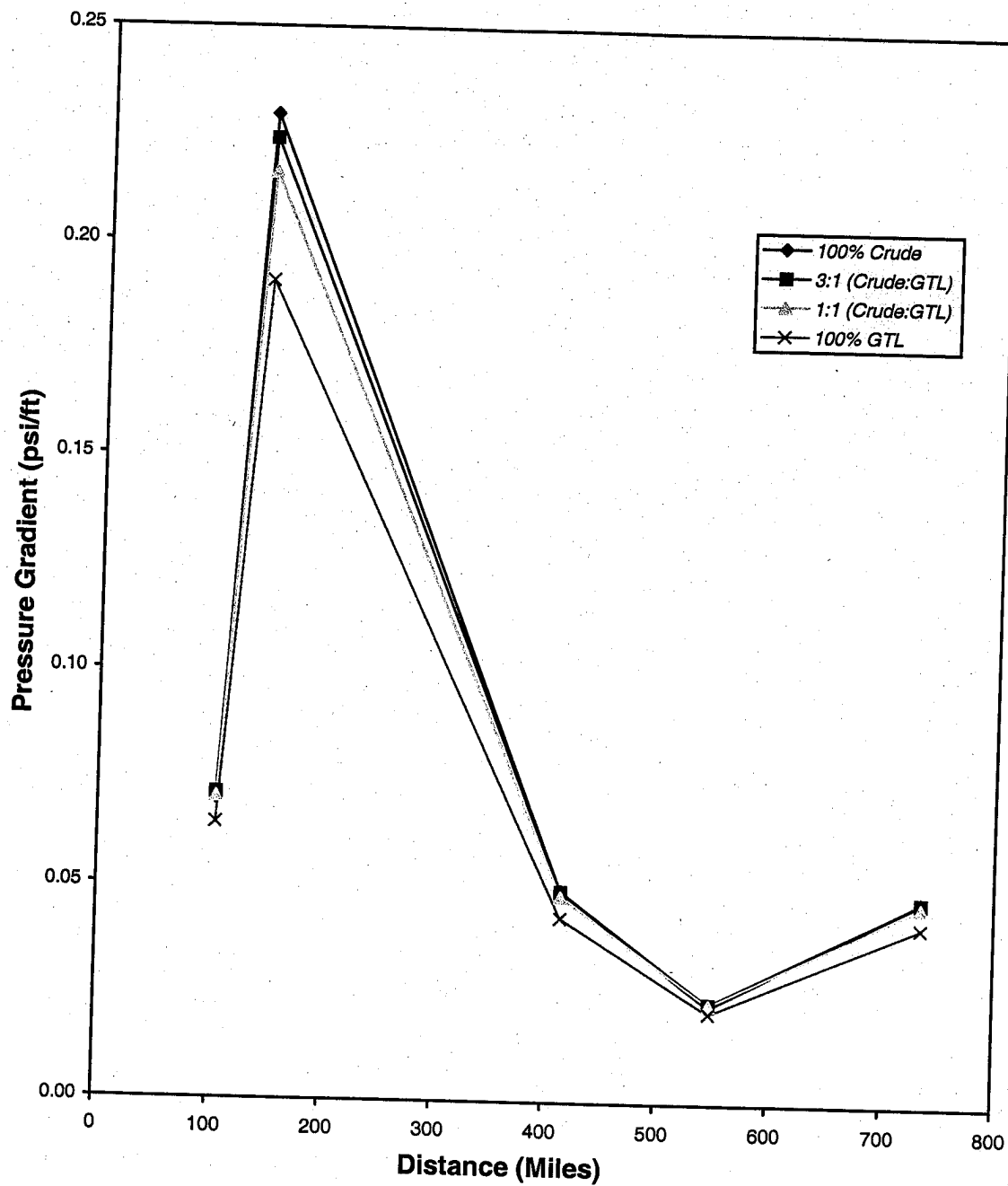
For commingled flow mode, a daily throughput of 1.1 MMBPD of total commingled fluid is considered. For a GTL to crude oil ratio of 1:1, the other input data are shown below.

Inlet Temperature = 90°F  
 Fluid: Specific Gravity = 0.833  
 Fluid specific gravity = 2.8 cp  
 Pipe Diameter = 48 inch  
 Pipe Roughness = 0.00001 ft

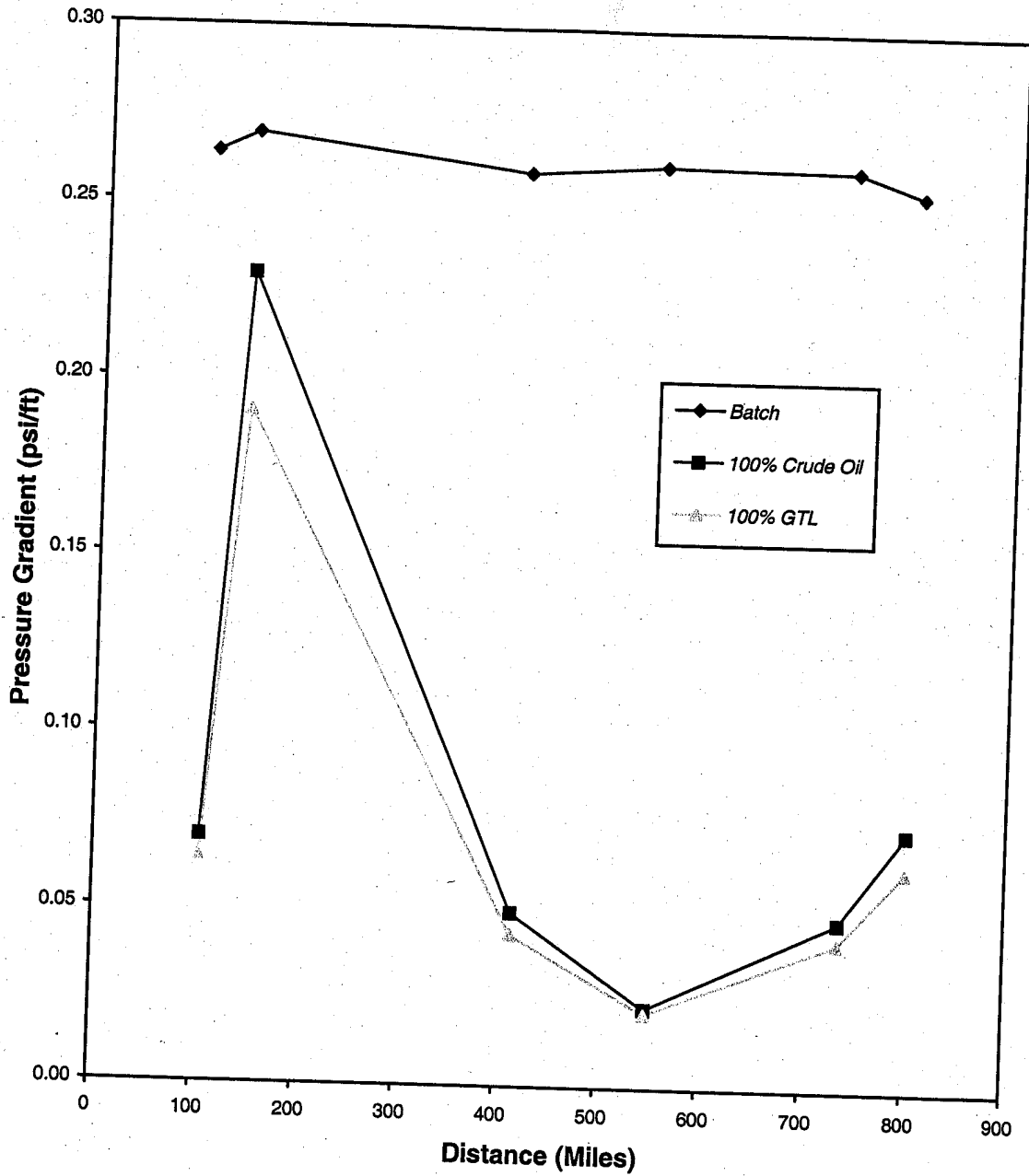
The sample calculations were also carried out for different blending ratios of GTL and Crude Oil. The ratios considered were:

- i. 100% Crude Oil
- ii. 75% Crude Oil + 25% GTL (3:1 ratio)
- iii. 50% Crude Oil + 50% GTL (1:1 ratio)
- iv. 100% GTL

The pressure gradients for commingled flow obtained from these computations are as shown in Figure 4.5. For comparison, pressure gradients from batch mode and commingled mode are plotted together in Figure 4.6. These results indicate that the pressure gradients obtained from the batch flow calculations are higher than those obtained from that of commingled flow. The reason for this difference is that for batch flow, the pressure gradient is the ratio of the total pressure drop across the slug to the slug length, whereas for commingled flow, it is the ratio of the total pressure drop to the length of the pipe segment.



**FIGURE 4.5** Pressure Gradient Plot for Commingled Flow



**FIGURE 4.6** Comparison Plot of Batch and Commingled Flow Modes

#### 4.6 CONCLUSIONS

The following conclusions are made, based on the results presented in this study.

1. Using the equations presented in this work, batch and commingled flow models can be analytically solved for predicting the pressure gradients encountered when considering the transport of GTL products and Crude Oil through the Trans-Alaskan Pipeline System (TAPS).

2. The derived flow equations presented here can be modified under specified operating conditions or constraints of the Trans-Alaskan Pipeline System (TAPS), using live GTL or Crude Oil data.
3. Mixing at the Oil-GTL interface in the case of batch mode transportation poses flow modeling and simulation difficulties.
4. The pressure gradients obtained from the batch flow calculations are higher than those obtained from that of commingled flow.

#### 4.7 NOMENCLATURE

$A$	cross-sectional area of the pipe, $m^2$ [ $ft^2$ ]
$C$	correction factor for the liquid hold-up in the slug
$C_e$	constant in Euler's equation
$C_o$	film distribution parameter
$D$	pipe diameter, m [inch.]
$D_f$	hydraulic diameter occupied by the film, m [inch.]
$E_{lf}$	liquid holdup in the film
$E_{ls}$	liquid holdup in the slug
$F_s$	resultant of forces
$f_f$	friction factor for the interface zone based on $R_{emz}$
$f_s$	friction factor in the liquid slug based on $R_{es}$
$g, g_c$	acceleration due to gravity, $9.81 \text{ m/s}^2$ or $32.2 \text{ ft/s}^2$
$h, z$	height or elevation, m [ft]
$h_f$	head loss due to friction, m [ft]
$\Delta h_p$	pump head, m [ft]
$k_m$	factor for the length of the mixing zone
$L$	length or distance, m [ft]
$l_m$	length of the mixing zone, m [ft]
$l_s$	length of the slug, m [ft]
$m_e$	mass exchange rate, $kg/s$ [ $lbm/s$ ]
$N_{Re}$	Reynolds number
$\Delta P$	pressure drop, $N/m^2$ [psi]
$\Delta P_a$	acceleration pressure drop, $N/m^2$ [psi]
$\Delta P_f$	frictional pressure drop, $N/m^2$ [psi]
$\Delta P_h$	hydrostatic pressure drop, $N/m^2$ [psi]
$\Delta P/L$	average pressure gradient, $N/m^2$ [psi]

$R_{emz}$	Reynolds number for the interface zone
$R_{es}$	Reynolds number for the liquid slug
$V_d$	drift velocity, m/s [ft/s]
$V_f$	Interface zone velocity, m/s [ft/s]
$V_m$	mixture velocity, m/s [ft/s]
$V_s$	average velocity of the slug, m/s [ft/s]
$V_{sl}$	superficial liquid velocity, m/s [ft/s]
$V_t$	transitional velocity, m/s [ft/s]
$W_p$	Liquid wetted perimeter of the pipe wall, m [inch]
$X$	Lockhart-Martinelli parameter
$\Delta z$	change in elevation, m [ft]
$\beta$	angle of inclination, °
$\varepsilon$	pipe roughness, m [ft]
$\mu_l$	Liquid viscosity, cp.
$\mu_{mz}$	Viscosity of the interface zone, cp.
$\rho_l$	liquid density, kg/m <sup>3</sup> [lb/gallon]
$\rho_{mz}$	Density of the interface zone, kg/m <sup>3</sup> [lb/gallon]
$\omega$	slug frequency

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