APPENDIX C

TRACER DATA ANALYSIS

Tracer Data Analysis

In analyzing tracer data, a minimum of two parameters is important for describing the residence time distribution of a pulse injected in a flow system (23,41): the first moment (μ) and the second moment (σ^2) The first moment is related to the mean residence time (τ) monitored between two reactor locations by the following equation:

$$\tau = \mu_2 - \mu_1 \qquad \text{(C-1)}$$

The first and second moments are determined from the following equations:

$$\mu = \int_0^\infty t\, c(t)\, d\, t / \int_0^\infty c(t) dt \qquad \text{(C-2)}$$

$$\sigma^2 = \int_0^\infty t^2 c(t)\, d\, t / \int_0^\infty c(t)\, d\, t - \mu^2 \qquad \text{(C-3)}$$

The integrals were evaluated on the computer numerically using Simpson's Rule. Error due to this numerical procedure should be minimal because of the very small (less than 0.2 sec) interval size used. However, calculation of moments using Equations C-2 and C-3 can result in significant errors due to error in concentration values at long times (41). This error can be minimized by using a modified method of analysis of moments (41). This method utilizes a Laplace transform of a concentration distribution:

$$c(s) = \int_0^\infty c(t) \exp(-st) dt / \int_0^\infty c(t)\, d\, t \qquad \text{(C-4)}$$

Differentiating with respect to s gives:

$$\frac{d^n}{ds^n}(c(s)) = -1^n \int_0^\infty t^n\, c(t) \exp(-st) d\, t / \int_0^\infty c(t)\, d\, t \qquad \text{(C-5)}$$

The modified first moment is found by calculating the slope of a plot of c(s) versus s as s approaches zero. The second moment is determined in a similar fashion.

Both methods were used to calculate the moments of the gas tracer concentration/time curves. The difference in the moments evaluated by the two different methods is an indication of the error in the concentration distribution at long times.

The gas linear velocity (V_g) between Locations 1 and 2 can be determined from the first moments by the following equation:

$$V_g = \frac{h}{\mu_2 - \mu_1} \qquad \text{(C-6)}$$

where h is the distance between Locations 1 and 2.

In addition, both first and second moments can be used to estimate parameters of a mixing model. Mixing of the gas in the system studied is characterized by the Peclet number:

$$\frac{1}{Pe_g} = \frac{E_g}{V_g h} \qquad \text{(C-7)}$$

where E_g is the dispersion coefficient.

A simple relationship between moments and the dispersion coefficient was given by Aris (43) as:

$$\frac{\Delta\sigma^2}{(\Delta\mu)^2} = \frac{2E_g}{V_g h} \qquad \text{(C-8)}$$

Although the results reported in this work are not sufficient to extend the work of other investigators (44) to three-phase fluidized systems, it may be worthwhile to estimate the order of magnitude dispersion coefficients for the systems studied in this work.