# CHAPTER VII

# CONCLUSIONS

This chapter summarizes the main results of the present experimental investigation, and presents the needed improvements to obtain high-accuracy measurements.

The Particle Tracking Velocimetry technique has been used for a three-dimensional, transient experimental study of single bubble dynamics in a restricted medium. The three-dimensional velocity field was reconstructed via stereoscopic matching of two-dimensional vectors from two camera images at different view angles. A hybrid tracking technique has been shown to be particularly effective to determine the flow around a bubble. The Spring Model tracking algorithm was successfully applied to describe the flow within the bubble's wake, while the ART-2 Neural Network was used to track the tracer particles in the liquid for four consecutive frames, mainly in regions away from the bubble. The ART-2 Neural Network provided data that were used for the Lagrangian evaluation of the velocity and acceleration required in the analysis of the forces acting on the bubble. The development of a Shadow Particle Image Velocimetry allowed the bubble's shape study, and an accurate measurement of the dimensions, orientation, trajectory, and velocity and acceleration of a bubble rising in water.

This study also has shown that the use of algorithms generally employed in computer vision would provide a great tool for flow visualization measurement techniques. Such algorithms include the Dynamic Generalized Hough Transform for bubble shape identification and reconstruction; the image processing techniques that allow for an accurate detection of boundaries and centroids of objects; and the epipolar geometry constrain, which is needed for the determination of the three world coordinates of a point inferred from the two stereo images.

In this investigation, 81 bubbles were sequentially released in stagnant water in a small-diameter pipe. The time interval between the release of the bubbles was in the order of few minutes. This time separation was enough to obtain quiescent flow between successive bubble injection. The flow around the bubble and within the wake was determined from ensemble averaging of instantaneous velocity fields. The ensemble average operation was performed by considering a conditional sampling technique. The condition of the averaging was that of specific bubble trajectory. The beginning of the trajectory is started once the bubble entered the viewing volume. The measurement volume was divided into 9×9×9 cartesian grid for the liquid nodalization scheme. The total number of cells was 729, although only 621 fall inside the pipe. The different bubble trajectories were determined by dividing the horizontal cross sectional plane into 5 zones, one of them positioned at the pipe center. The Z-direction was divided in 4 regions.

The velocity fields were determined for each specific bubble trajectory. In order to better quantify the wall influence on the flow structure, and to obtain more samples for the averaging operation, the flow field was assumed symmetrical for each nodal region that had the similar bubble trajectory. Therefore, these flow fields were combined. The data of the regions in the pipe annulus were added to the third quadrant ($x < 0$ and $y < 0$) in the horizontal plane, due to the assumption of the symmetry of the flow field. A reasonable number of samples for the average for bubble trajectories close to the wall could be performed.

The bubble Reynolds number was in the range from 350 to 700. In the present study, the bubbles rose in a helical path, in agreement with the map of Clift *et al.* (1978). Rocking motion was also observed, with both the PIV and Shadow cameras. The ellipsoidal shape of a bubble is a result of the pressure difference inside and outside of the bubble. By combining the influence of the seed particles and the collision frequency with the pipe wall, the final bubble shape was an oblate spheroid. The largest dimension was always in the Y-direction, followed by the semiaxis in the Z-direction, and the smallest semiaxis was in the X-direction. It is noted that bubbles rising at the pipe center zone had the maximum value of the semimajor axis, while the bubbles rising close to the wall had larger semiaxes in the X and Z directions. This is in agreement with the fact that the wall influence is to elongate the dimensions of the semiaxes parallel to the wall, while it diminishes the dimension of the semiaxis perpendicular to the wall. The average eccentricity of the bubbles was found to be well predicted by a correlation available in the literature.

Regarding the bubble motion, it was found that the inclusion of the flow disturbance in the bubble motion equation generates scattering of the data for the drag and lift coefficients.

In Chapter I, a list of the objectives of this experimental study was presented. Each objective accomplishment is addressed next.

*How much energy does the bubble bring into the test volume?* For the two bubble trajectories studied, bubbles rising along the pipe center and others close to the wall, the total kinetic energy in the test volume shows similar characteristics. A sharp increase in the total kinetic energy exists at the first time step the bubble enters the viewing volume ($t_p$ = 0 ms). The maximum is reached at the third time step the bubble is present in the test zone ($t_p$ = 33.33 ms). This maximum is about 70 μJ. The decrease of the total kinetic energy is noticeable at the first time step the bubble has departed the measurement volume ($t_a$ = 0 ms). After the third time step bubble departed the viewing volume, at $t_a$ = 33.33 ms, the behavior is rather constant. This time also indicated the departure of the primary wake from the viewing volume. The bubble trajectory close to the wall shows higher total kinetic after $t_a$ = 33.33 ms.

The higher total kinetic energy of the bubble rising close to the wall is a result of the turbulent kinetic energy being generated. The mean kinetic energy is higher for the bubble trajectory at the pipe center until $t_a$ = 33.33 ms. After that time both trajectories have similar values of the mean kinetic energy.

Larger mean kinetic energy is produced by bubbles rising along the pipe core, (before $t_a$ = 33.33 ms), due to the acceleration of the liquid surrounding the bubble to compensate the reduction of flow area available. However, in the case of the bubble trajectory close to the wall, the liquid more than one bubble diameter away from the bubble in the horizontal plane, at same height, does not accelerate as much.

*How long does it take for the viscosity and wall friction to dissipate the energy in the viewing volume?* After the primary wake completely leaves the measurement zone ($t_a$ = 33.33 ms), the transient behavior of the total kinetic energy is practically constant for both bubble trajectories. This indicates that the decay constant is more than 4.0 μJ/s.

*How much turbulence does the bubble induce during its rising path?* The bubbles rising close to the wall generate more turbulent kinetic energy. When the bubble is within the test volume, the turbulent kinetic energy is less than the mean until $t_a$ = 16.67 ms. At this period the turbulent kinetic energy is about 40% of the total for the trajectory close to the wall; while it is 35% for the trajectory along the pipe center. After $t_a$ = 33.33 ms, more than 70% of the total kinetic energy in the measurement volume is the contribution of turbulent kinetic energy.

*How is the structure of the wake?* The rocking and spiraling bubble motion was predominant in this experiment. Therefore, a combination of continuous trailing wake and vortex shedding is expected. However, due to the averaging process and low spatial resolution, the wake structure could not be accurately described. The results obtained here seem to indicate the wake structure associated to a spiraling bubble, more than a rocking one, particularly for the primary wake. For the far wake, it is probable that the irregular vortex structures observed were originated from vortex shedding, and then associated to rocking bubble motion. The vortex structures shed from the bubble's surface did not spread for a long distance due to the wall effect. Further, the interactions of these vortex spots with the wall, wake/wake interactions, and wake/mean flow interactions, distort the circulatory motion.

The primary wake of a bubble rising in the pipe core seems to be closed and not turbulent. This could not be completely determined because of the lack of resolution in the measurements. For the bubble trajectory close to the wall, the primary wake was distorted and of smaller volume than that of the case of bubble trajectory along the pipe core. It was not clear to conclude whether the wake is closed or open.

For the bubbles rising along the pipe core, the flow closer to the bubble travels upstream surrounding the bubble's primary wake, and tilts and collides at the end of the wake, close to the center of the pipe. This collided flow generates a flow barrier zone, which decelerates any flow passing through it. The flow barrier was also generated in the case of bubbles rising close to the wall but was restricted to a zone of an area on the order of the bubble size. Also, the flow in the radial direction apart from the bubble with a distance more than one bubble diameter is practically undisturbed, reducing its magnitude, as it gets closer to the pipe wall.

*How far upstream and downstream the bubble is the bubble-induced agitation felt?* The influence zone of the bubble is different for the two trajectories. For trajectory along the pipe core, the bubble influence reached downstream to an average distance of two bubble diameters, measured from the bubble's top. For the bubble trajectories close to the pipe wall, the bubble influence downstream reached a longer distance. This average distance was about three bubble diameters. The increase in the distance is due to acceleration increase of the liquid flow between the bubble surface and the pipe wall, which is limited with the wall friction effect. This friction overcomes the momentum transfer from the bubble to the liquid. Consequently, the bubble pushes up liquid that experiences this resistance. Subsequently, liquid layers are then pushed further up.

The bubble's primary wake for trajectories along the pipe center extends three bubble diameters upstream, from the bubble's bottom. The primary wake length for trajectories close to the wall is extended to a distance of about three bubble diameters.

*What is the magnitude of the different forces acting on the bubble?* Only the drag and lift forces were evaluated in this study, because one of the objectives is to determine the drag and lift coefficients. In addition, the results presented here were obtained under the assumption of the liquid flow field was quiescent, and without the inclusion of the acceleration parameter. It is noted that the measured lift force, and therefore the lift coefficient, has contributions from the Saffman and Magnus effects. The first effect exhibits from the shear flow generated by the flow acceleration around the bubble plus the flow deceleration because of wall friction. The Magnus force arises from the rocking motion of the bubble. The average drag force for the bubbles rising in the trajectory in the pipe core was $9.2\times10^{-5}$ N, while for the trajectory close to the wall the average drag force was $9.3\times10^{-5}$ N. The lift forces were smaller, as expected. For the trajectory close to the wall, the average lift force was $4.0\times10^{-5}$ N, while for the trajectory along the pipe center was $3.4\times10^{-5}$ N.

*What is the pipe wall influence on the drag and lift coefficients?* The wall influence on these coefficients is actually introduced through the velocities and accelerations of the liquid and the bubble. The results show that the seed particle presence in the water has a significant influence on the velocities of the bubbles, due to the changes of the fluid properties (seeds contaminate the water). By using data for contaminated systems, there was a difference of only about 8% with the measured average bubble velocities. This value can be attributed to direct wall influence, and it very closely agrees with the 7% predicted from a commonly used correlation.

In this investigation, the instantaneous drag coefficient did not show any obvious trend with respect to the rotation parameter. This phenomenon has been also found for large and small Reynolds numbers in other experiments. The instantaneous drag coefficient shows a similar behavior with the Reynolds number as the standard drag curve. The results presented next were obtained under the assumption of the liquid flow field was quiescent, and without the inclusion of the acceleration parameter. The drag coefficient for the bubble rising in the pipe core was 0.90; while it was 0.98 for the trajectory close to the wall. These results reflect the decrease of velocity due to wall friction. However, these values are higher than the values obtained from different correlations commonly used in the literature. It was surprising to find out that the drag coefficient calculated using the terminal velocity condition was indeed very close to that computed through force balance. The difference in values was from 5 to 10%. The bubble terminal velocity was no achieved in this experiment.

Regarding the lift coefficient, no trend was found as function of the Reynolds number and rotation parameter, of the instantaneous lift values. For the trajectory close to the wall it has an average value of 0.44. For the bubble rising in the pipe center, the value was 0.37. The theoretical value commonly used in the literature is 0.5. This value is for spherical particles in inviscid fluid. The difference between the theoretical value and the present experimental results arises from the bubble shape, the wall influence, and the rocking motion of the bubble. The oblate spheroid shape indicates an asymmetry of the pressure field around the bubble. In addition, the rocking motion of the bubble implies that the bubble experience different velocity at different locations of the bubble surface. Consequently, the pressure field changes to accommodate the velocity variations across the bubble. These pressure field variations induce lateral forces on the bubble. The lift force that an ellipsoidal bubble experiences is different from that exerted over a spherical bubble.

Observe that the values given above are an average. The instantaneous values were both higher and smaller that the average. This confirms that the pressure field depends on the instantaneous shape and translation and angular velocities of the bubble.

In general, there still exist some aspects that need investigation. It is clear that higher resolution is required to describe the flow surrounding the bubble. Particularly, it is necessary to determine the structure of the flow close and within the bubble's boundary layer, and inside the wake. Moreover, it is recommended to study the bubble Reynolds number within a small range, to better quantify the drag and lift coefficients for ellipsoidal bubbles. Clearly, a large sample of the instantaneous data is needed to determine the accurate mean and turbulent patterns. The liquid and the bubble grids of the test volume should be increased. The bubble trajectory needs to be followed longer, so the average operations yield more accurate data, and Lagrangian statistics can be achieved.

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