TABLE I

CORRELATIONS FOR THE EXPANSION OF LIQUID FLUIDIZED BEDS

Author	Solids	Equation	Range of Applicability
Richardson and Zaki (6)	Spherical particles	$\epsilon_1^{\,n} = U_1/U_t$	
		where:	
		$n = 4.65 + 20\ d/D$	$Re_t < 0.2$
		$n = (4.4 + 18\ d/D)Re_t^{-0.03}$	$0.2 < Re_t < 1.0$
		$n = (4.4 + 18\ d/D)Re_t^{-0.1}$	$1 < Re_t < 200$
		$n = 4.4\ Re_t^{-0.1}$	$200 < Re_t < 500$
		$n = 2.4$	$Re_t > 500$
Richardson and Zaki (6)	Glass or steel cylinders and steel plates	$\epsilon_1^{\,n} = U_1/U_t$	
		$n = 2.7\ K^{0.16}$	$Re_t > 500$
		where:	
		$K = (\pi/6)d_s^{\,3}/d_p^{\,3} = \left(\frac{\pi}{4}\right)^{5/2}\left(\frac{d}{l}\right)^{1/2}$ (for a cylinder)	
Fouda and Capes (8)	Irregularly shaped particles	$(1 - K'\epsilon_s)^n = U_1/U_t$	
		where:	
		n = same as in Richardson-Zaki correlation	Same as in Richardson-Zaki correlation
		$K' = 1$ for spheres	
		$K' = 1.17$ to 1.43 for equi-dimensional bud irregularly shaped particles	
Barnea and Mizrahi (9)	Spherical particles	$C_{DM} = \left(0.63 + \frac{4.8}{\sqrt{Re_m}}\right)$	$10^{-3} < Re_m < 3 \times 10^4$
		where:	
		$Re_m = Re\left(\frac{U_r/U_t}{\exp(5\epsilon_s/3(1-\epsilon_s))}\right)$	
		$C_{DM} = C_0\left(\frac{U_r}{U_t}\right)^2\left(\frac{1-\epsilon_s}{1+\epsilon_s^{1/3}}\right)$	
		$\frac{U_r}{U_t} = \frac{1}{1+\epsilon_s^{1/3}}$	

(Table Continued)

TABLE I

CORRELATIONS FOR THE EXPANSION OF LIQUID FLUIDIZED BEDS

-2-

Author	Solids	Equation	Range of Applicability
Wallis (10)	Spherical particles	$j_1^* = 2/9\ r^{*2} \epsilon_1^{4.7}$	$Re_m < 1.5$
		$j_1^* = 0.307\ r^{*1.21} \epsilon_1^{3.47}$	$1.5 < Re_m < 100$
		$j_1^* = 0.693\ r^{*0.858} \epsilon_1^{2.92}$	$100 < Re_m < 1080$
		$j_1^* = 2.5\ r^{*1.2} \epsilon_1^{2.35}$	$1080 < Re_m$
		where:	
		$j_1^* = j_1 \left(\frac{\rho_1}{\mu_1 g\ (\rho_s - \rho_1)} \right)^{1/3}$	
		$r^* = \left(\frac{\rho_1 g (\rho_s - \rho_1)}{\mu_1} \right)^2$	

TABLE II

EXPERIMENTAL CONDITIONS FOR BUBBLE COALESCENCE

Investigator	Liquid	Gas	Solid	Solid Dimension (mm)	Bed Diameter (mm)	Character of Bed
Ostergaard (17)	Water	Air	Sand	0.64	250 x 250	Coalescing
Massimilla, et al. (18)	Water	Air	Sand	0.22,0.26	86 x 61	Coalescing
	Water	Air	Glass Beads	0.79,1.09	86 x 61	Coalescing
Viswanatan, et al. (19)	Water	Air	Quartz Particles	0.649,0.928	50.8	Coalescing
	Water	Air	Glass Beads	4.0	50.8	Non-Coalescing
Adlington and Thompson (2)	Water	-	Sand	3.0	76.0	Coalescing
	White Spirit	-	Alumina	0.3-3.0	76.0	Coalescing
Lee (21)	Water	Air	Glass Beads	6	-	Non-Coalescing

TABLE III

SUMMARY OF DATA FOR GAS/LIQUID/SOLID FLUIDIZATION

Investigators	System	Particle Size	Experimental Unit	Parameters Studied	Measured Quantities
1) Razumov, Manshilin, Nemets(22)	Air/Water/Sand Air/Water/Glass Beads	0.493 to 1.27 mm	300 mm Diameter Column	U_g, U_l	ϵ_s
2) Michelsen, Ostergaard(23)	Air/Water/Glass Beads	1,3,6 mm	152 mm Diameter Column	U_g, U_l, Particle Size	ϵ_s, ϵ_l
3) Dakshinamurty, Subramanyam, Rao(24,25)	Air/Water/Various Spherical Particles Air/Kerosene/Various Spherical Particles	1.06 to 6.8 mm	56 mm Diameter Column	Particle Size and Density σ, U_g, U_l	ϵ_s
4) Viswanathan, Kakai, Murti(19)	Air/Water/Quartz Particles Air/Water/Glass Beads	0.928,0.649 mm 4 mm	50.8 mm Diameter Column	U_g, U_l, Particle Size	ϵ_l, ϵ_g
5) Kim, Baker, Bergougnou(11)	Air/Water/Glass Beads Air/Water/Irregular Gravel	6 mm 2.6 mm	660x25 mm Rectangular Channel	U_g, U_l	ϵ_l, ϵ_g, Bubble Size and Velocity
6) Kim, Baker, Bergougnou(26)	Air-Water/Acetone-Glass Beads Air-Water/Acetone-Irregular Gravel Air-Sugar/Water-Glass Beads Air-Carboxymethyl Cellulose/Water-Glass Beads Air-Sugar/Water-Irregular Gravel Air-Carboxymethyl Cellulose/Water-Irregular Gravel	1.6 mm 2.6 mm 1.6 mm	660x25 mm Rectangular Channel	μ_l, σ, U_l, U_g	ϵ_g, ϵ_l
7) Blum and Toman(27)	Nitrogen/Light Mineral Oil	3.2x3.2mm	102 mm Diameter Column	U_l, U_g, μ_l	ϵ_s
8) Nemets, Razumov, Manshilin(28)	Air/Water/Sand Air/Heptane/Sand Air-Water/Clycerol-Sand	0.820 mm	90 mm Diameter Column	U_l, U_g, μ_l	ϵ_l, ϵ_g
9) Bruce, Revel-Chion(29)	Air/Water/Glass Spheres	2,4,6,8 mm	46.3 mm Diameter Column	U_l, U_g	ϵ_s, Bubble Size
10) Ostergaard,	Air/Water/Glass Ballotini	0.28 to 2.2 mm	51 and 102 mm Diameter Column	U_l, U_g	ϵ_s

TABLE IV

EMPIRICAL CORRELATIONS FOR THREE-PHASE BEDS

Author	Correlation	Gas/Liquid	Solids (Dimension)	Column Diameter or Dimensions (mm)	Comments
Kim, Baker, Bergougnou(11)	$(\epsilon_l)_{U_g=0} - \epsilon_l = 0.0025 \left(Fr_l \frac{\rho_s}{\rho_l}\right)^{0.149} \left(Fr_g \frac{\rho_s}{\rho_s}\right)^{0.161} \left(\frac{Re_l}{Re_g}\right)^{0.289}$	Air/Water	Glass Beads (6 mm)	660 x 25	Empirical, no consideration for bed contraction
	$(\epsilon_l)_{U_g=0} = 0.409(Fr_l \rho_s/\rho_l)^{0.193}(Re_l)^{0.074}$				
Kim, Baker, Bergougnou(26)	$(\epsilon_l+\epsilon_g) = 1.40(Fr_l)^{0.17}(We)^{0.078}$ (Expanding Beds)				Empirical for expanding and contracting beds
	$(\epsilon_l+\epsilon_g)_{U_g=0} = 1.3(Fr_l)^{0.139}(We)^{0.073}\exp[0.31U_l/U_g(\epsilon_l)_{U_g=0}]$ (Contracting Beds)	Air/Sugar Solutions Air/Carboxylmethyl Cellulose Solution Air/Water Acetone	Glass Beads (6,1 mm) Irregular Gravel (2.6 mm)	660 x 25	
	$(\epsilon_l)_{U_g=0} = 1.353(Fr_l)^{0.206}(Re_l)^{-0.082}(We)^{0.092}$				
Dakshinamurty, Subrahmanyam, Rao (24,25)	$(\epsilon_g+\epsilon_l) = \left(\frac{U_l}{KU_t}\right)^{m}\left(\frac{\mu_l U_g}{\sigma}\right)^{n}$ $n = 0.08$ $K = 2.22,\ m = 0.41,\ Re_t < 500$ $K = 2.65,\ m = 0.6,\ Re_t > 500$	Air/Water Air/Kerosene	Numerous Diameters 1.06 to 6.8 mm	56 mm Diameter	Empirical, no consideration for bed contraction
Blum, Toman(27)	$\frac{(\epsilon_g+\epsilon_l) - (\epsilon_l)_{U_g=0}}{1-(\epsilon_l)_{U_g=0}} = f(U_g)$	Nitrogen/Light Mineral Oil	Cylinders Dia., mm — Length, mm 3.2 — 3.2 4.8 — 2.4 4.8 — 4.8	102	Empirical, no consideration for bed contraction
Razumov,	$\epsilon_s = 0.578 - 3.198\ U_l - 0.538\ U_g$ $\epsilon_l = 0.422 + 0.135\ U_l/\overline{d}^{0.54} - 1.82\ U_g$ $\epsilon_g = K(1-\epsilon_s)^{0.20}(U_g/U_l)^{0.79}$	Air/Water	Sand, Slag Beads 0.49 to 1.27 mm	300 mm	Empirical, no consideration for bed contractions

TABLE V

BHATIA-EPSTEIN MODEL

Equations

1) Tabulation of $\epsilon_k''/\epsilon_g''$ vs. ϵ_g (Letan and Kehat*).

2) $\epsilon_l = \epsilon_k(1 - X_k) + \epsilon_l''_f(1 - \epsilon_g - \epsilon_k + X_k\epsilon_k)$

3) $\epsilon_c + \epsilon_g + \epsilon_l = 1$

4) $U_g = V_g\epsilon_g$

5) $\epsilon''_{l_f} = \frac{U_l - V_g\epsilon_k(1 - X_k)}{U_t(1 - \epsilon_g - \epsilon_k)}^{1/n}$

6) $V_g = \frac{u_l + U_g + \epsilon_l''_f(1 - \epsilon_g - \epsilon_k)V_{gl}'''}{1 - \epsilon_c}$

V_{gl} = relative velocity between bubble phase and liquid in particulate phase

7) $\epsilon_k = \frac{\epsilon_g\epsilon_k''}{\epsilon_g''}(1 - \epsilon_c)^3$

8) $V_{gl}''' = U_{t_a} + 2\ U_g$

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*Letan, R., and E. Kehat, AIChE J, 14, 396, 1969.

TABLE VI

PHYSICAL PROPERTIES OF KEROSENE

Temperature, °C	24	38	65
Viscosity, cp	1.39	1.15	0.80
Surface Tension, Dynes/cm	28.6	--	24.6
Density, gm/cc	0.79	0.78	0.77

TABLE VII

CUMULATIVE SIZE DISTRIBUTION OF COAL CHAR

CUMULATIVE NUMBER AND NUMBER % GREATER THAN STATED SIZE

Size, μm	Cumulative Number	Cumulative Number %
0	2916	100
1.1	2635	90.4
2.7	1921	65.9
3.8	1396	47.9
5.4	922	31.6
8.1	529	18.1
13.5	212	7.3
18.7	115	3.9
29.7	64	2.2
51.3	24	0.8
72.8	13	0.4
94.4	8	0.3

TABLE VIII

COMPARISON OF COAL CHAR WITH H-COAL REACTOR FINES

	Coal Char	Recycle Oil Solids (Pyridene Insolubles)
Particle Density, g/cc	1.7	2.3
Density Distribution	90% 0.8-2.2 g/cc. Even distribution.	85% 1.2-2.8 g/cc. Even distribution.
Particle Size	70%--325 Mesh.	90%--325 Mesh.

Source of Information: Bernard, R. F., "Cold Flow Model Study of Reactor Fluid Dynamics--Topical Report, Task III," DOE Contract EX-77-C-01-2547, HRI Report No. FE-2547-15, March, 1978.

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TABLE IX

PROPERTIES OF HDS-2A CATALYST

Nominal Diameter: 1.6 mm
Nominal Length: 4.8 mm

Particle Density, g/cc	1.0794
Pore Volume, cm/g	0.77
Surface Area, m^2/g	296
Length Distribution (mm)	
Below 3.8	1
3.8-4.0	0
4.0-4.2	2
4.2-4.4	6
4.4-4.6	9
4.6-4.8	5
4.8-5.0	4
5.0-5.1	7
5.1-5.3	9
5.3-5.5	4
5.5-5.7	0
5.7-5.9	1
5.9-6.1	3
6.1-6.3	4
Over 6.3	5
Average	5.1

TABLE X

COAL CHAR DISTRIBUTION ALONG THE REACTOR: HDS-2A CATALYST, L = 4.8 MM, D = 1.6 MM

Test No.	Fines* Vol%	Slurry Flow Cm/Sec	Gas Flow Cm/Sec	Concentration in Wt%						
				Reactor Bottom	152 Cm	305 Cm	457 Cm	Recycle Line	Separator Feed	Average
204-23	5.0	5.5	1.1	7.4	7.5	6.7	11.9	--	--	8.4
205-26	10.0	3.0	7.6	14.5	14.7	16.4	16.7	--	--	15.6
206-05	15.0	4.3	0	21.8	22.0	21.8	20.5	20.3	16.3	21.5
206-(2)	15.0	3.0	4.6	21.8	24.6	25.2	25.2	23.3	22.2	24.2
206-67	15.0	3.0	6.1	23.9	24.4	25.9	24.8	23.9	22.6	24.7
205-45	11.5	3.0	3.0	17.2	17.2	17.2	17.2	--	--	17.2

*This is a calculated number based on charged amount to slurry preparation tank. Wt% = 1.5 vol%.

TABLE XI

PARTICLE SIZE DISTRIBUTION OF REACTOR COAL CHAR SAMPLES

Slurry Concentration = 15 Vol%
Liquid Velocity, Cm/Sec = 0.10
Gas Velocity, Cm/Sec = 4.6

CUMULATIVE NUMBER AND CUMULATIVE NUMBER PERCENT
GREATER THAN STATED SIZE

	Reactor Bottom Sample AU77-13		457 cm Level Sample AU77-16	
Size, μm	Cumulative Number	Cumulative Number %	Cumulative Number	Cumulative Number %
0	4409	100	4142	100
1.1	3588	81.4	3220	77.7
2.7	2485	56.4	2109	50.9
3.8	1971	44.7	1632	39.4
5.4	1472	33.4	1148	27.7
8.1	1017	23.1	714	17.2
13.5	542	12.3	316	7.6
18.9	291	6.6	164	4.0
29.7	113	2.6	61	1.5
51.3	32	0.7	8	0.2
70.2	5	0.1	4	0.1
91.8	2	0.05	2	0.05
Max. Size, μm	-------- 108.0 --------		-------- 118.6 --------	
Avg Size, μm	-------- 3.5 --------		-------- 2.8 --------	

TABLE XII

VARIATION IN RICHARDSON-ZAKI INDEX

Run No.	Test No.	Temp, °C	Fines, Vol%	Index n
201	9-44	24	0	2.9
203	1-11	24	1	3.2
204	1-28	24	5	3.4
205	1-34	24	10	3.5
205	37-64	24	11.5	3.5
206	1-56	24	15.0	3.6
206	57-74	24	16.5	4.8
208	1-31	65	16.5	3.7
209	1-30	38	16.5	3.5

TABLE XIII

SOLUTION OF THE BHATIA-EPSTEIN MODEL

1) Select initial values of ϵ_c, ϵ_l, ϵ_g.
2) Compute: $V_{g_1}''' = U_{t_\theta} + 2\,U_g$.
3) Compute: $\epsilon_k''/\epsilon_g''$ - values given by Letan and Kehat.
4) Compute: $\epsilon_k = \epsilon_g \dfrac{\epsilon_k''}{\epsilon_g''}(1 - \epsilon_c)^3$
5) Compute: $V_g = U_g/\epsilon_g$
6) Compute: $\epsilon_l''_f$ from Richardson-Zaki relationship
7) Compute: $V_g = \dfrac{U_l + U_g + \epsilon_l''_f (1 - \epsilon_g - \epsilon_k)V_{g_1}'''}{1 - \epsilon_c}$
8) Compute: $\epsilon_{l_2} = \epsilon_k(1 - X_k) + \epsilon_{l_f}''(1 - \epsilon_g - \epsilon_k + X_k\epsilon_k)$
9) Compute new ϵ_g: $\epsilon_{g_2} = U_g/V_g$
10) Average: $\epsilon_{g_1} + \epsilon_{g_2} = \epsilon_{g_{avg}}$
11) Compute new ϵ_c from ϵg_{avg} and ϵ_{l_2}

Iterate until $\Delta\epsilon_g$ $\Delta\epsilon_l$ are small.

Figure 1

H-Coal PDU reactor

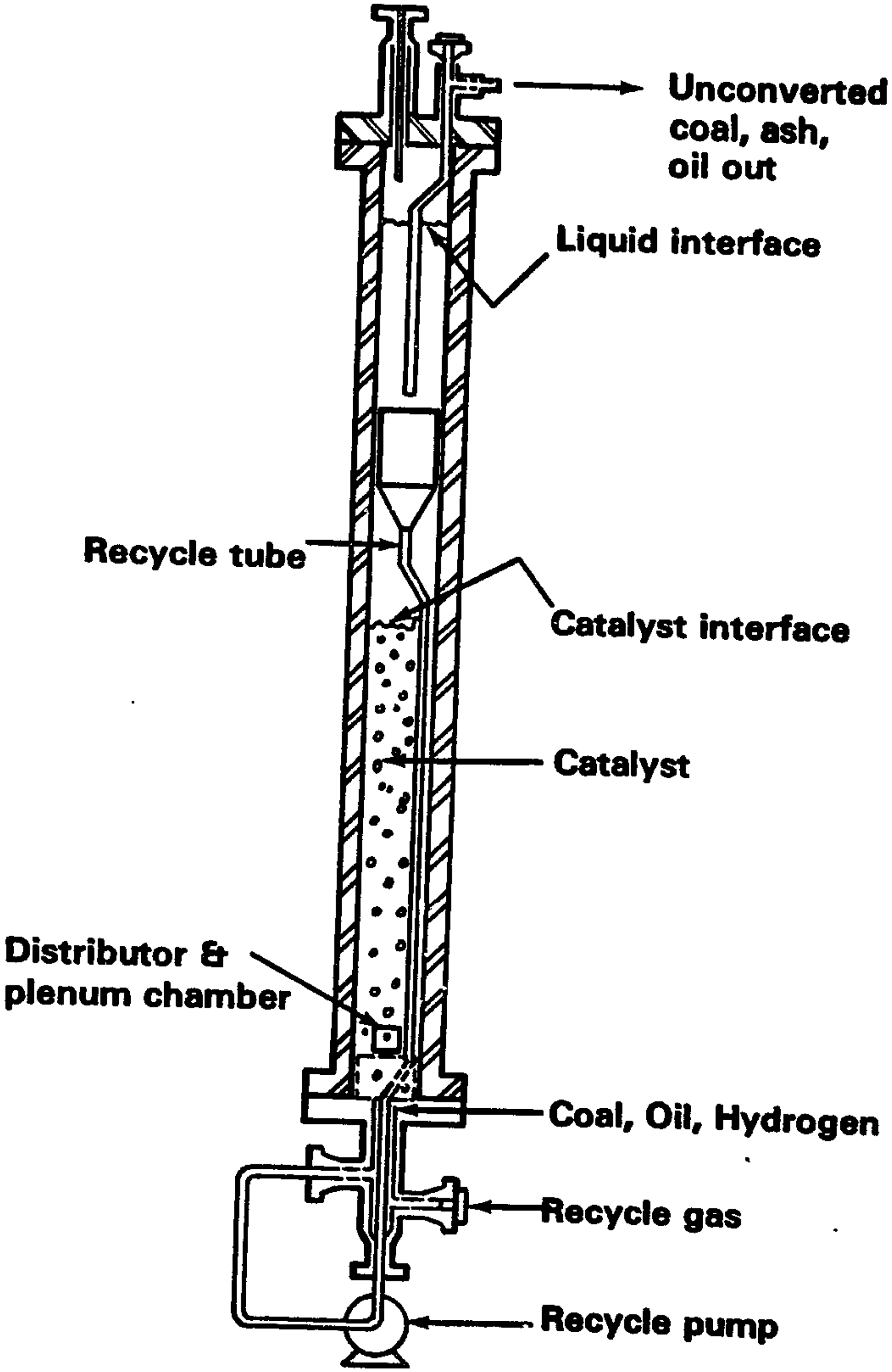

Figure 2

H-Coal process development unit

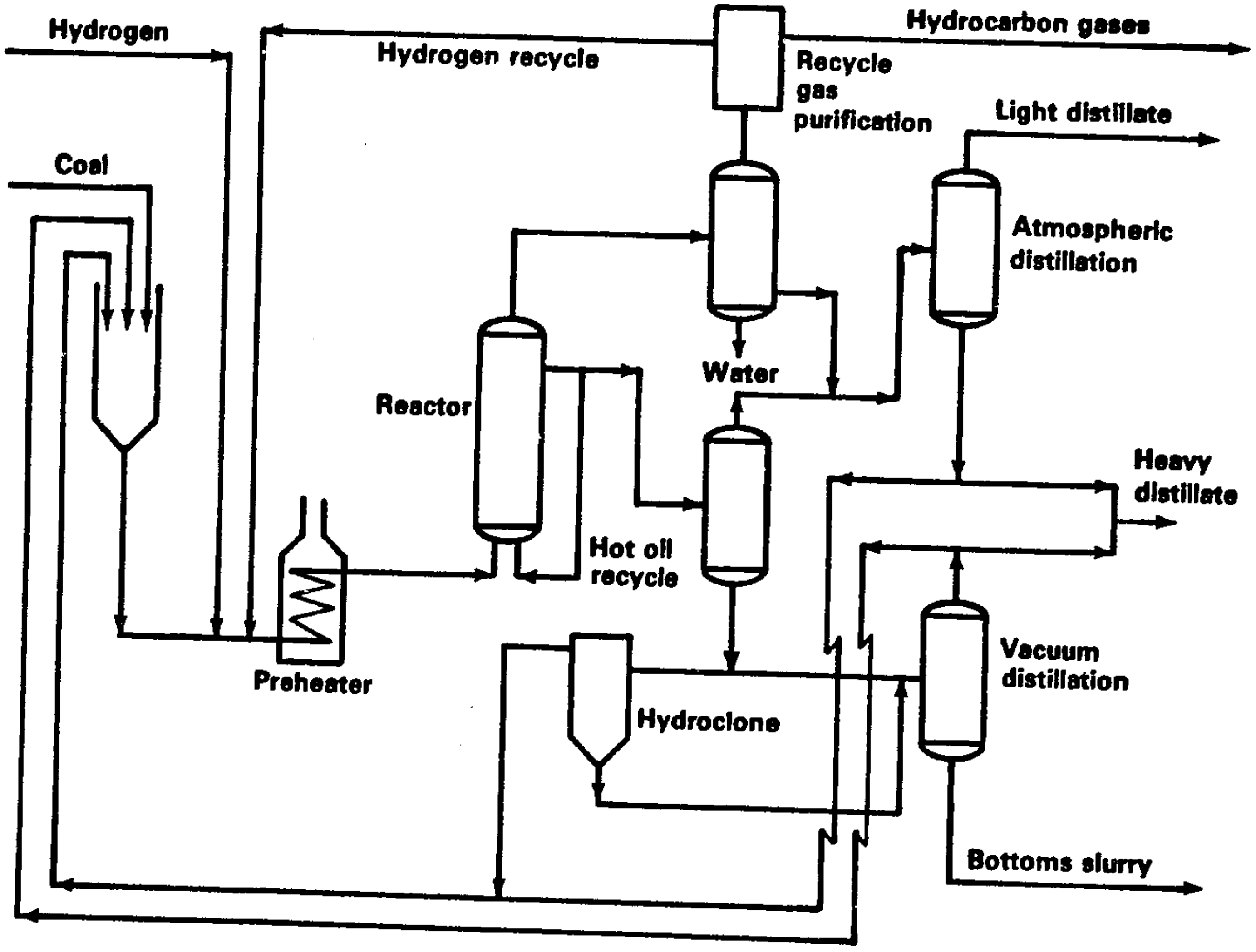

Figure 3

Drift flux vs. gas holdup: Darton and Harrison (5)
(Gas/liquid/solid fluidization)

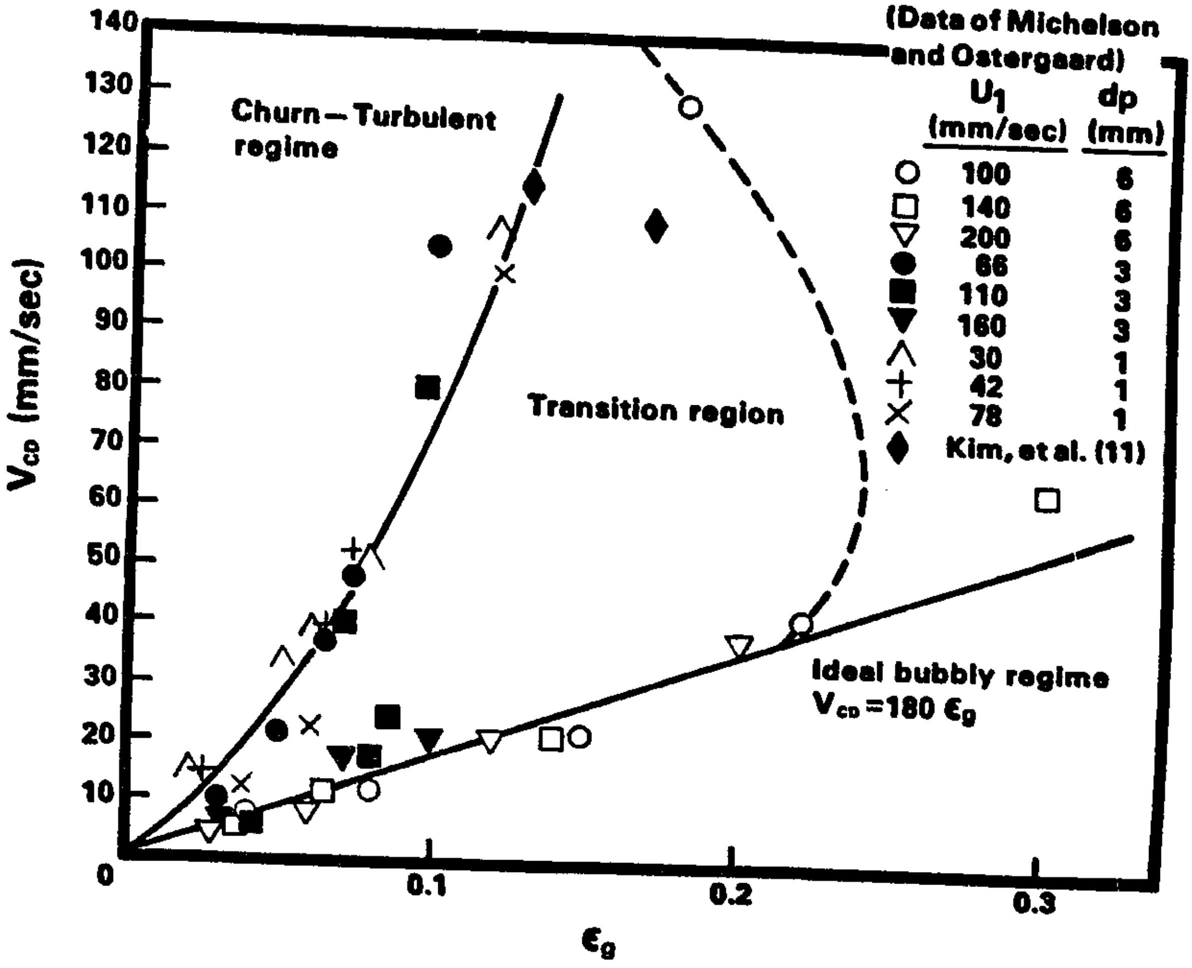

Figure 4

Schematic diagram of the fluid dynamics unit

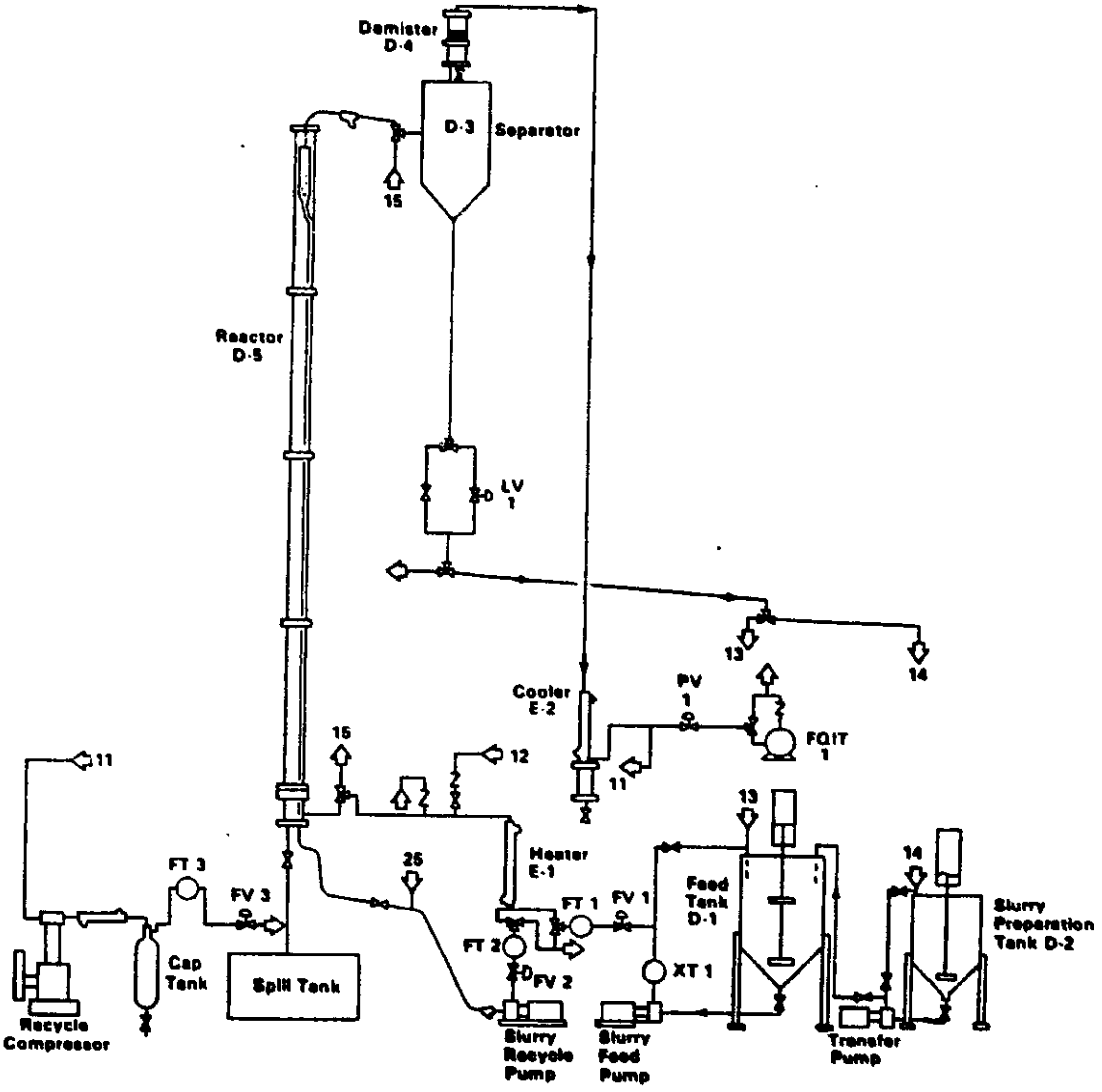

Figure 5

Reactor design