

buoyant force on a bubble, which increases its velocity relative to the liquid and thereby tends to decrease the gas volume fraction. However, greater liquid density can also increase bubble breakup, which reduces bubble size and velocity relative to the liquid and thereby tends to increase the gas volume fraction. A majority of the empirical gas-liquid correlations in the exhaustive review by Joshi *et al.* (1998) indicate that increasing liquid density generally increases gas volume fraction. This suggests that the bubble breakup effect of ρ_L tends to dominate in bubble column reactors. Similarly, greater liquid viscosity decreases bubble velocity but increases bubble size; in this case, the latter effect dominates and the net result is generally a reduction in gas volume fraction. Greater surface tension generally increases bubble size and reduces gas volume fraction as well. These opposing trends with liquid properties are supported by the common dependence of gas volume fraction upon the Bond and Archimedes numbers in empirical correlations (see, for example, Eqs. 2.11 and 2.12).

The experimental values in Figures 5.13 and 5.14 were fit using a dimensional density-based correlation:

$$\bar{\epsilon}_G = \frac{0.0555U_G^{0.549}}{[\rho_L(1 - \bar{\epsilon}_S^{NOM}) + \rho_S\bar{\epsilon}_S^{NOM}]^{1.017}}, \quad (5.9)$$

where U_G is in cm/s, ρ_L and ρ_S are in g/cm³, $\bar{\epsilon}_S^{NOM}$ is the nominal solid volume fraction, and $\bar{\epsilon}_G$ is the average gas volume fraction. The bracketed term in this correlation is the density of an effective “dense-phase” medium composed of the liquid and the solid. For the glass beads and the 400 μm polystyrene beads, the correlation generally reproduces the data to within the experimental uncertainty of ± 0.02 (Figures 5.18 and 5.19). The data from the experiments with 200 μm polystyrene particles are not fit well by the correlation, however, suggesting that other physical phenomena (such as particle diameter or surface tension) must be considered in describing the behavior of the gas phase. Note also that the trend of decreasing gas volume fraction with increasing medium density runs counter to most gas-liquid correlations, as explained in the previous paragraph. Further work to parameterize the data and compare results to existing two-phase correlations is recommended.

5.4. Conclusions

Material distributions have been measured successfully in many three-phase bubble-column flows using the combined EIT/GDT technique. The experiments involved both glass particles with a density substantially greater than the liquid density and polystyrene particles that were nearly neutrally buoyant. The gas volume fraction radial profiles were found to be relatively insensitive to the particle loading for all of the particle types examined, although the glass particles appeared to produce lower gas volume fractions at higher solids loadings. For solid volume fractions up to 0.30, the observed variation of gas volume fraction distribution with particle density and diameter was no more than ± 0.04 . While the observed variations can be explained with some accuracy by an effective gas/dense phase model, other physical mechanisms not yet accounted for may also be involved. Further analysis of the data will be

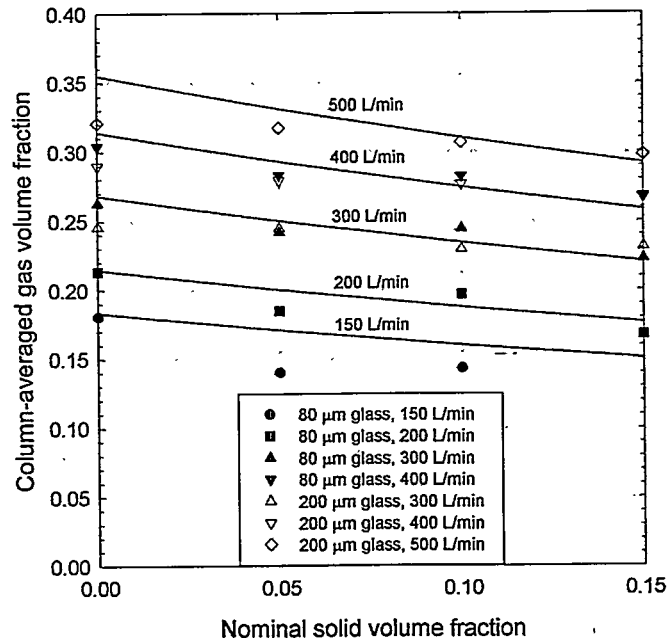


Figure 5.18. Comparison of measured and predicted average gas volume fractions in three-phase flows involving glass particles. Symbols are measured values; curves are predicted by Eq. 5.9.

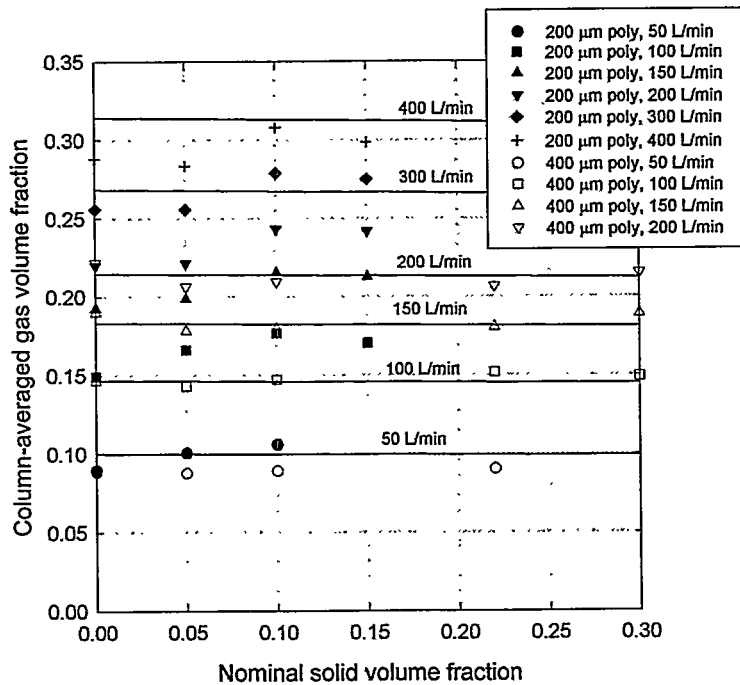


Figure 5.19. Comparison of measured and predicted average gas volume fractions in three-phase flows involving polystyrene particles. Symbols are measured values; curves are predicted by Eq. 5.9.

performed and may determine whether the effective two-phase model is an adequate description of three-phase flows.

With respect to bubble-column hydrodynamics, several conclusions may be drawn from these results about the changes (or lack of changes) to gas-liquid flows with the addition of a solid phase. For all solids loadings and gas flow rates examined, the time-averaged gas volume fraction profiles were still found to be roughly parabolic, indicating (along with visual observations) that the flow was still in the churn-turbulent regime. No dense layer of the solid phase was observed along the sides of the vessel; the particles appear to be approximately uniformly distributed in the liquid. Similarly, small but observable changes in the dense phase ratio $\bar{\epsilon}_s / (\bar{\epsilon}_s + \bar{\epsilon}_L)$ with superficial gas velocity U_G suggest that at lower gas flow rates, the solid phase is not homogeneously distributed in the liquid phase, as assumed in some simplified three-phase models. However, as the gas flow rate increases and lofting efficiency is improved, the dense phase ratio approaches values determined from the homogeneous assumption.

6. Conclusions

The application of tomographic techniques to multiphase flows continues to be a significant research topic, where goals include improvements in temporal and spatial resolution and quantitative accuracy. In a contribution to this field, an electrical-impedance tomography (EIT) system, developed collaboratively by Sandia National Laboratories and the University of Michigan, has been validated for quantitative measurements of solid volume fractions in solid-liquid flows and gas volume fractions and radial profiles in axisymmetric gas-liquid flows. Many of these validation studies were performed through comparison of EIT results to measurements from an established gamma-densitometry tomography (GDT) system. In solid-liquid flows, EIT was used to measure solid volume fractions up to 0.05, with excellent agreement between EIT results and the nominal values determined from the mass of solids in each flow. Very good agreement was also obtained between EIT and GDT in those solid-liquid flows when the latter technique could be applied. Air-water bubble-column flows in a 19-cm ID bubble column were also measured with both EIT and GDT. For cross-sectionally-averaged gas volume fractions up to 0.15, the average gas volume fraction values and radial profiles from EIT and GDT agreed to within the EIT measurement uncertainty of ± 0.01 , even with large radial variations across the column. The Maxwell-Hewitt relations used to convert conductivity information to conducting phase distributions were found to be accurate for cases where the assumptions inherent in their derivation were valid. A minor, physically reasonable modification of the Maxwell-Hewitt relations was found to improve the level of agreement between GDT and EIT.

After validation of the EIT method, the EIT and GDT systems were successfully combined to accurately measure radial phase distributions in a series of three-phase, solid-gas-liquid bubble-column flows resembling those in slurry bubble-column reactors. The goal of these experiments was to examine the effect of the solid phase on the hydrodynamic behavior of the remaining phases, representing another step toward the application of EIT to industrial multiphase flows. The study employed solid phases with conductive properties similar to air but densities on the same order of magnitude as water, so that each of the three phases involved a unique combination of attenuating and conductive properties. This also required that the constitutive equations for both measurement methods be solved simultaneously to determine the distribution of all three phases. Four particle types were examined: 200-700 micron polystyrene spheres (specific gravity 1.04), 40-100 micron glass spheres (specific gravity 2.4), 120-200 micron glass spheres, and 170-260 micron polystyrene spheres. Solid volume fractions of up to 0.30 were examined over superficial gas velocities in the 2-30 cm/s range. This approach of repeating the experiments with solid particles of different diameters and specific gravities was taken to determine the influences of these quantities on radial phase profiles.

Over the range of solid volume fractions from 0 to 0.30, the gas distribution for each gas flow rate was relatively insensitive to the amount of solids present in the mixture. For all of the particle types that were examined, a maximum variation in gas volume fraction of ± 0.04 was

observed for a given superficial gas velocity. Neutrally buoyant particles were found to have almost no effect on the gas distribution. A slight decrease in gas volume fraction with nominal solid volume fraction was observed with the glass particles, which were substantially more dense than the liquid. The physical mechanism responsible for this effect is not yet clear. Particle diameter was found to have only a small effect on gas distribution. By comparison, surface-tension changes from contaminants added with the particles were more important than changes from the presence of the particles themselves. Finally, no dense layer of particles along the side walls of the vessel was observed; the particles appear to be approximately uniformly distributed in the column. Results such as these are essential to the development of accurate phase interaction models for three-phase flows. The implication of the present results for flow modeling is that an effective two-phase description may be accurate for describing the three-phase flows studied here.

The results of the bubble column experiments indicate that the assumption of a parabolic or quartic phase distribution is reasonably accurate when information is averaged over time periods as short as twenty seconds. Since the existing EIT system can clearly determine time-averaged phase distributions in a short time, the question of its potential to measure instantaneous phase distributions arises. For nearly instantaneous measurements, data must be acquired over periods approaching the time scale of the turbulent two-phase flow, estimated for the conditions of Chapters 4 and 5 to be on the order of a few milliseconds. Even with the recent conversion to a Pentium computer to control the EIT system, a single projection set cannot be obtained this quickly. In theory, the minimum time required for an AC electrical instrument to acquire a single data point is two cycles of the excitation voltage, so that the maximum data acquisition rate is half the excitation frequency. Based on a sixteen electrode system operating at 50 kHz, and assuming the minimum number of voltage values required for a reconstruction is given by Eq. 3.3, an instantaneous measurement of the phase distribution could theoretically be obtained in 5 ms, comparable to the flow time scale. However, the uncertainty of such a reconstruction must be considered. Early experiments with Lexan "phantom" inclusions similar to those in Chapter 3 suggest that ten projection sets with 1920 data points each is the minimum necessary to produce a reconstruction of a stationary distribution with acceptably low uncertainty. Tests using the quartic reconstruction algorithm in Chapter 4 suggest that 100 projection sets are required for accurate reconstructions of phase profiles in dynamic flows. Thus, while the present system can produce time-averaged phase profiles quickly and accurately, time-resolved information in dynamic flows appears to be beyond its capabilities.

Future efforts will focus on two areas. The first area involves improvements to the EIT system described in this work. As this report is being completed, the EIT system is being integrated into the LabView environment to standardize data acquisition. The time necessary to acquire projection sets may be reduced by as much as an order of magnitude with improved PC hardware, and measurement accuracy can be increased by a conversion of data acquisition hardware from 12-bit to 16-bit precision. The second area of interest involves extensions of the EIT technique to Sandia's slurry bubble-column reactor (SBCR) testbed and to other industrial-scale systems (*e.g.*, conducting vessels with insulating liquids). With the successful combination of EIT and GDT reported here, additional investigations are now possible to help develop three-phase flow models for bubble-column reactor design. Three-phase experiments in the 48.26-cm ID SBCR testbed using the LabView EIT and GDT systems are planned upon completion of a sparger-parameter study currently under way. It has also been noted that the existing EIT

hardware could be modified to collect voltage projections from flows in which the phases are predominantly capacitive. Rather than modify the existing system, consideration is being given to a new capacitive tomography system that would determine three-dimensional solids distributions in a dynamic gas-solid flow. The new system would be evaluated in an industrial-scale, gas-solid reactor recently constructed at Sandia. Other proposed enhancements (of lower priority) to the existing EIT system include a reconstruction algorithm that would resolve non-axisymmetric phase distributions, and the combination of two or more separate EIT systems to measure phase velocities.

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