CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

(1) A semi-theoretical model for determining the rate of bubble coalescence in turbulent gas-liquid dispersions has been developed, based on the theories of molecular collision and isotropic turbulence, and an empirical (or semi-theoretical) coalescence efficiency function of the coalescence time and the interaction time. This model has only one unknown parameter and has included the effect of size ratio between two bubbles. This model may be used to the liquid-liquid dispersions. Unfortunately, no experimental results are available to verify this model.

(2) A theoretical bubble or drop breakup rate model has been proposed, based on the principles of molecular collision, isotropic turbulence and probability. This breakage rate model has none unknown parameters or constants. In addition, it does not need to assume a daughter bubble or drop size distribution like most of previous work. The daughter size distribution can be obtained directly from this model. The predicted daughter bubble or drop size distributions by this model have shown very good agreement with the experimental results of Hesketh *et al.* (1991a).

(3) The parallel film concept has been extended to develop two theoretical models for the approach between two equal or unequal sized fluid particles. A simple model gives expressions for the film area and the interaction time. The

latter has been used to determine the coalescence efficiency in the bubble coalescence rate model mentioned above. A more general model can only give numerical solutions for the film area and the interaction time. The predicted film areas by the general model are in good agreement with the experimental data of Scheele and Leng (1971), but those by the simple model are not good. Both models give similar results for the interaction time and do not coincide with the experimental data well. The detailed analysis for the data shows that the deviations may be caused by the oscillations of the particles themselves.

(4) A one-dimensional population balance model has been proposed for determining local bubble size distributions in bubble columns. In this model, the coalescence and breakup rate models are used. This model has an unknown parameter and the predicted results above the entrance region for the air-water system in a tall bubble column by the model are in agreement with the measured results by the five-point conductivity probe technique, especially for high superficial gas velocities. This model shows that, for the air-water system, the bubble size distribution above the entrance region of the column is not sensitive to the bubble sizes formed by the gas distributor.

(5) The pseudo-homogeneous fluid concept, which has been frequently used for modeling the liquid circulation in bubble columns but has also been often mixed with the two-fluid concept, has been clarified by this work. An analytical expression for the radial liquid velocity profile has been derived from this concept, using a radial turbulent viscosity distribution in single-phase pipeline flows and an empirical expression for the radial gas holdup distribution in bubble columns.

A new two-fluid model for the liquid circulation has been proposed. Unlike the previous two-fluid models, the assumption that the shear stresses in liquid phase and bubble phase are equal, is not used, because this assumption is found to be unreasonable not only in physical meaning but also in the same equation of motion as the pseudo-homogeneous concept. The new two-fluid model can also give analytical expression for the liquid velocity profile, using the same distributions for turbulent viscosity and gas holdup as in the pseudo-homogeneous model.

These two models are easy and fast to use, and are in good agreement with the experimental data reported in the literature. Since these models have not ignored the effect of molecular viscosity, they can also give good predictions even for high viscosity liquids. The pseudo-homogeneous model is simpler but needs to tune the gas holdup distribution parameter, *m*, to the experimental data. The two-fluid model is better in physical concept and usually need not to tune the parameter, *m*.

(6) A new model for determining bubble sizes and specific interfacial areas in bubble columns by the dynamic gas disengagement technique is developed, based on a concept of non-uniform steady state distribution of the bubble dispersion. Interpreting the non-uniformity in the axial direction, this model using the DGD technique gives axial gas holdup distributions, and by assuming an axially homogeneous dispersion, a radial gas holdup distribution can be obtained. The Sauter mean diameters and specific interfacial areas for several gas-liquid systems have been estimated by the model and the DGD technique. The results in the air-water system are compared and are in agreement with those measured by a five-point conductivity probe technique. The obtained axial gas holdup distributions agree well with those measured by Menzel (1989) and the radial gas holdup distributions for the same system are also in reasonable agreement with those measured in this work. Results are also reported for the air-salt water, air-aqueous propanol and air-dodecylbenzene systems.

(7) By using the five-point electrical conductivity probe technique, measurements for bubble size distributions, bubble velocities and movement directions, and local gas holdups in the air-water system have be done. These data have been used verify the modeling work.

8.2 Recommendations

(1) The coalescence rate model needs to be improved. It is better to avoid using the empirical function, $exp(-t_C/t_I)$, and to employ the probability method, $t_I \geq t_C$,

for determining the coalescence efficiency. Another consideration for improvement may be to model the collision and the drainage processes simultaneously (they are intrinsically synchronous), instead of separating them into two different processes as done at present.

In order to verify modeling work for coalescence rate and coalescence efficiency, experimental data for these are required. but unfortunately they are not available. The experimental data for the interaction time between two unequal sized bubbles or drops are also required, as to confirm the effect of size ratio.

(2) More general models for the coalescence time between two equal or unequal sized particles with fully- or partially- mobile interfaces need to be developed. The interaction time model also needs to be improved, since the parallel film concept may have large deviation from the fact when size difference between two bubbles or drops is large.

(3) Two-dimensional population models or one-dimensional models with consideration of backmixing may need to be developed. More experimental measurements about the bubble size distribution and gas holdup at gas distributors are needed.

(4) In almost all modeling work for the liquid circulation in bubble columns, the empirical gas holdup distribution, $\varepsilon_{G0}(1-\varphi^m)$, has to be used and the parameter, m, needs to be known or tuned. This empirical function should be avoided or the parameter, m, should be determined by modeling. This may be done utilizing the principle of energy minimization or assuming analogy of the turbulent properties between single-phase and multi-phase flows.

(5) Theoretical or semi-theoretical relationships for bubble terminal velocity in wide ranges need to be developed. Modeling and measurement for gas holdup and bubble size distribution near the gas distributor are also needed.

(6) The effect of solid particles should be considered into all the above modeling work, in order to enable them useful to three-phase systems.

NOMENCLATURE

Latin Letters

A	dimensionless film area (in Chapter 4), $(r/R_1)^2$; or dimensionless parameter defined in Equation (6.12)
A_c	column cross-section area, m^2
A_{max}	maximum dimensionless film area
a	specific interfacial area, 1/m
B	dimensionless parameter defined in Equation (6.12)
BC	birth rate for bubbles by coalescence, 1/m^3/s
BB	birth rate for bubbles by breakage, 1/m^3/s
c	dimensionless coefficient in Equation (6.9)
c_f	coefficient defined in Equation (3.21)
c_1	adjustable constant in coalescence probability model
c_2, c_3	constants defined by Equations (3.15) and (3.17)
D	column diameter, m
DC	death rate for bubbles by coalescence, 1/m^3/s
DB	death rate for bubbles by breakage, 1/m^3/s
d_e	equivalent bubble diameter based volume, m
$\tilde{d}_e$	dimensionless equivalent bubble diameter, m
d_h	diameter of holes on the gas distributor, m
d_I, d_{II}	diameters of bubble I and II, m
d_i, d_j	diameters of bubble i and j, m
d_s	Sauter mean bubble diameter, m
E	energy spectrum function of turbulence, m^3/s^2
$E_{k,avl}$	available kinetic energy defined by Chesters (1993), J
$E_{k,int}$	internal kinetic energy, J
e	energy of individual eddies, J

$\bar{e}_i$	increase in surface energy due to a bubble breakage, J
F_c	capillary force acting within film, N
F^*_I, F^*_{II}	dimensionless forces defined by Equation (4.32), Equation (4.39)
F_{ji}	interaction force that phase j acts on phase i per unit dispersion volume, N/m^3
F_r	drag force acting on particles, N
F_s	interfacial drag force between gas and liquid phases per unit dispersion volume, N/m^3
F_1, F_2	external forces on fluid particles 1 and 2, N
f_{BV}	breakage volume fraction
g	gravitational acceleration, m/s^2
H	dispersion height, m
H_0	dispersion height at steady state, m
H_i	dispersion height at the end of period i, m
H_n	static liquid height, m
I_q	functions defined by Equations (6.16) and (6.17), $q = 0, 1, 2, \cdots$
i, j	referring bubbles, probe sensors, disengagement periods and flow phases (gas and liquid)
k	wave number of eddies in turbulence, 1/m
k_d	wave number of eddies of viscous dissipation, $k_d = (\varepsilon/\nu_L{}^3)^{1/4}$, m^{-1}
k_e	wave number of eddies of energy-containing, 1/m
k_R	Reichardt's constant in Equation (6.8)
L_c	central vertical chord length of bubble, m
L_d	vertical distance between the bubble leading surface at the centerline and at the radial position, x_p, m
L_e	vertical length of bubble at radial position of x_p, m
L_0	vertical length of bubble at radial position of x_p, m
Mo	Morton number, $g\mu_L{}^4(\rho_L-\rho_G)/(\rho_L\sigma^3)$
m	shape factor for gas holdup distribution in Equation (6.9)
m_{eq}	equivalent mass of two particles defined by Equation (4.10), kg
m_1, m_2	effective mass of particles 1 and 2, defined by Equation (4.9), kg
N_h	number of holes on the gas distributor
n	number of bubble classes

n_i, n_j	number densities (per unit dispersion volume) of bubbles i and j, $1/m^3$
n_λ	eddy number density (per unit dispersion volume) for eddies between λ and $\lambda+d\lambda$, $1/m^4$
P	pressure of continuous phase, Pa
P_B	conditional breakage probability
P_C	coalescence efficiency
p_e	distribution density function of turbulent kinetic energy, 1/J
Q	dispersion flow rate, m^3/s
Q_G	gas flow rate, m^3/s
Q_L	(net) liquid flow rate, m^3/s
R	column radius, m
$\tilde{R}$	dimensionless radius, $R\hat{u}/\nu_L$
R_{eo}	equivalent radius defined by Chesters and Hofman (1982), m
R_1, R_2	radii of particles 1 and 2, R_1 refers to the smaller one, m
Re_0	maximum liquid Reynolds number, Du_{l0}/ν_L
r	film radius (Chapter 4), m; or radial position in columns, m
S_f	Surface force number, defined by Equation (4.33)
S_i	stress force acting on phase i per unit dispersion volume, N/m^3
Δs	increase of particle surface due to deformation, m^2
s_i	slope of fitted straight line i, m/s
$\boldsymbol{T}_i$	stress tensor acting on phase i per unit volume of i, N/m^2
T_i	pulse duration time for probe sensor i caused by bubble, s
T_i^m	modified pulse duration time (Burgess and Calderbank, 1975), s
t	time, s
t_C	coalescence time, s
$t_{d,i}$	time delay after disengagement period i, t_i^m-t_i, s
t_I	interaction time, s
t_i	delay time for probe sensor i in 2, s; or time at end of disengagement period i, s
Δt_i	disengagement time for period i, t_i - t_{i-1}^m, s
t_{max}	duration time when the film area reaches the maximum, s
t_{total}	total time in a measurement, s

$\bar{u}$	vector velocity of dispersion. m/s
$\hat{u}$	friction velocity, $(\lvert\tau_W\rvert/\rho_L)^{1/2}$, m/s
u_b	bubble rise velocity, m/s
$u_{b,a}$	absolute velocity of bubble motion, m/s
$u_{b,z}$	bubble velocity in the vertical direction, m/s
u_{cm}	velocity of center of mass of a two-particle-system, m/s
u^*_{cm}	dimensionless velocity of center of mass of a two-particle-system
u_{cm0}	velocity of center of mass of a two-particle-system at $t = 0$, m/s
u_G	superficial gas velocity, m/s
u_g	gas phase velocity, m/s
u_h	gas velocity in the holes, m/s
u_i	rise velocity of bubble class i, m/s; or velocity vector of flow phase i, m/s
$\bar{u}_i$, $\bar{u}_j$	velocities of bubbles i and j in turbulence, m/s; or apparent rise velocities in periods i and j, m/s
$\bar{u}_{ij}$, u_{ij}	approach velocity between bubbles i and j in turbulence, m/s
u_L	liquid superficial velocity, m/s
u_l	liquid phase velocity, m/s
$\bar{u}_l$	dimensionless liquid velocity, u_l/u_{l0}
u_{l0}	liquid velocity at central axis, m/s
u_r	approach velocity between the centers of mass of two particles, m/s
u^*_r	dimensionless approach velocity, $u_r(\rho_c R_1/\sigma)^{1/2}$
u_{r0}	approach velocity between the centers of mass of two particles at $t = 0$, m/s
u_s	slip velocity, m/s
u_t	bubble terminal velocity, m/s
$\bar{u}_t$	dimensionless terminal velocity
u_1, u_2	velocities of particles 1 and 2 relative to liquid, m/s
u_{10}, u_{20}	velocities of particles 1 and 2 relative to liquid at $t = 0$, m/s
$\bar{u}_\lambda$	eddy velocity, m/s
v_{Gi}	disengaged volume of bubble class i, m^3
v_I, v_{II}	volumes of daughter bubbles I and II
v_i, v_j	volumes of bubbles i and j, m^3
v_1, v_2	velocities for particles 1 and 2, defined in Equation (4.2), m/s

We	Weber number of particles, defined by Equation (4.14)
We_i	Weber number defined in Equation (3.26)
We_{ij}	Weber number defined in Equation (3.10)
W_i	body force acting on phase i per unit dispersion volume, N/m^3
X	spatial vector, m
x_p	horizonal distance between probe sensors 0 and 4 in the design of Burgess and Calderbank (1975), m
x_{pi}	horizonal distance between probe sensors 0 and i, m
y_{pi}	vertical distance between probe sensors 0 and i, m
y_{pi}^{m}	modified vertical distance between probe sensors 0 and i, m
z	axial position along the column from the gas distributor, m; or distance between the mass centers or the geometrical centers of particles (Chapter 4), m

Greek Letters

α	universal constant in turbulence, usually $\alpha = 1.5$ (Tennekes and Lumley, 1972); or dimensionless parameter defined by Equation (6.11)
β	constant defined in Equation (3.3); or the parameter determining excess pressure in film (Chapter 4); or dimensionless parameter defined by Equation (6.11)
β	universal constant in turbulence, $\beta = (3/5)\Gamma(1/3)\alpha$ (Batchelor, 1953)
$\Gamma()$	gamma function
γ	virtual mass coefficient
γ_{eff}	effective coefficient of virtual mass (Chesters and Hofman, 1982)
δ	film thickness, m or unit tensor
ε	energy dissipation rate per unit mass, m^2/s^3
ε_G	local gas holdup; or averaged gas holdup over the cross-section (Chapter 5)
$\bar{\varepsilon}_G$	average gas holdup over the cross-section
ε_{Gi}	void fraction of bubble class i
$\bar{\varepsilon}_{Gi}$	total void fraction in disengagement zone at the top in period i

ε_{G0}	gas holdup at the column axis
ε_L	local liquid holdup, $\varepsilon_L = 1-\varepsilon_G$
ε_v	energy dissipation rate per unit volume, kg/m/s^3
$\eta()$	breakage kernel or daughter bubble size distribution, 1/m^3
θ	angle between the probe axis and the bubble trajectory, °; or phase angle at contact of drops, °
χ	dimensionless energy in energy distribution function
χ_c	dimensionless critical breakage energy
λ	coefficient defined by Equation (4.13); or eddy size, $\lambda = 2\pi/k$, m
λ_d	eddy size of viscous dissipation, m
λ_e	eddy size of energy-containing, m
λ_1-λ_4	coefficients defined by Equations (4.29)-(4.31) and (4.38)
μ_c	viscosity of continuous phase, Pa·s
μ_L	liquid viscosity, Pa·s
ν_L	liquid kinematic viscosity, m^2/s
ν_t	liquid eddy kinematic viscosity, m^2/s
ξ	size ratio between an eddy and a bubble, $\xi = \lambda/d_i$; or size ratio R_1/R_2 in Chapter 4
ξ_i	axially non-uniform distribution coefficient for bubble class i
ξ_{ij}	size ratio between bubbles i and j, $\xi_{ij} = d_i/d_j$
ρ_c	density of continuous phase, kg/m^3
ρ_d	density of dispersed phase, kg/m^3
ρ_G	gas density, kg/m^3
ρ_i	density of phase i, kg/m^3
ρ_L	liquid density, kg/m^3
σ	surface tension, N/m
τ	dimensionless time defined in Equation (4.17) or local shear stress, N/m^2
τ_i	shear stress vector of phase i, N/m^2
τ_{max}	maximum dimensionless time
τ_w	shear stress at the wall, kg/m/s^2
φ	horizontal angle, defined in Figure 2.5, °
φ_i	angle of probe sensor i, defined in Figure 2.4, °
o	dimensionless radial position in columns, r/R

$\Omega_B(v_i;v_j)$	breakup rate for bubbles of size v_i that may form bubbles of size v_j, 1/m^3/s
$\Omega_B(v_i)$	breakup rate for bubbles of size v_i, 1/m^3/s
$\Omega_C(v_i;v_j)$	coalescence rate between bubbles of sizes v_i and v_j, 1/m^3/s
ω_C	collision frequency between bubbles, 1/m^3/s
$\omega_{b,\lambda}$	bombarding frequency on a bubble surface by eddies of size between λ and $\lambda+d\lambda$, 1/m^4/s

Superscripts

m	modified
inlet	near or at the distributor
i, j	referring bubbles, probe sensors, disengagement periods and flow phases (gas and liquid)
*	non-dimensionlization
–	non-dimensionlization