# INTRINSIC FLOW BEHAVIOR IN A SLURRY BUBBLE COLUMN UNDER HIGH PRESSURE AND HIGH TEMPERATURE CONDITIONS 

Quarter Report
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## Simulation of Single Bubble Formation in Liquid-Solid Suspensions at High Pressures with Pressure Fluctuations in the Gas Chamber

## Highlights

- The behavior of bubble formation from a single orifice connected to a gas chamber in a non-aqueous liquid and a liquid-solid suspension at high pressures (up to 8.3 MPa ) was investigated. An optic fiber probe was used to measure the initial bubble size in liquids and liquid-solid suspensions.
- A two-stage spherical bubble formation model was extended to describe the bubble formation process in liquid-solid suspensions with pressure fluctuations in the gas chamber. The effects of particle and pressure on bubble motion and pressure balance at the gas-liquid interface were taken into consideration in the model through various forces.
- The model can reasonably predict the initial bubble sizes obtained in this study, as well those reported in the literature under high-pressure conditions. The predicted trends of pressure and particle effects on initial bubble size under various bubble formation conditions also agreed well with the experimental observations.
- The model simulation showed that pressure in the gas chamber and gas flow rate through the orifice change significantly with time during bubble formation. Even for bubble formation under constant flow conditions, there still exist fluctuations in pressure in the gas chamber and in gas flow rate through the orifice at the beginning of bubble formation.


## Work Conducted

## Model Equations

## Bubble Motion Equation

The motion equation of a rising bubble in liquid-solid suspensions can be described based on a balance of all the forces acting on the bubble (Luo et al., 1998). The forces induced by the liquid include the upward forces [effective buoyancy $\left(F_{B}\right)$ and gas momentum $\left(F_{M}\right)$ ], and the downward resistance [liquid drag $\left(F_{D}\right)$, surface tension force $\left(F_{\sigma}\right)$, bubble inertial force $\left(F_{I, g}\right)$, and Basset force $\left(F_{\text {Basset }}\right)$. Two other downward forces on the bubble account for the particle effect on bubble motion, i.e., the particle-bubble collision force $\left(F_{C}\right)$ and the suspension inertial force $\left(F_{I, m}\right)$, due to acceleration of the liquid and particles surrounding the bubble (Luo et al., 1998). Therefore, the overall force balance on the bubble formed in liquid-solid suspensions can be written as

$$
\begin{equation*}
F_{B}+F_{M}=F_{D}+F_{\sigma}+F_{\text {Basset }}+F_{I, g}+F_{C}+F_{I, m} . \tag{1}
\end{equation*}
$$

The expansion and detachment stages follow the same force balance equation [Eq. (1)], although the expressions for the same force in two stages may be different. The expressions for all the forces under two stages are given in Table 1. The particle-bubble collision force is merely the rate of momentum change of particles colliding with the bubble surface. The suspension inertial force can be derived from the suspension flow field around an accelerating bubble. Luo et al. (1998) quantified the flow field of the suspension around a two-dimensional bubble by using the Particle Image Velocimetry (PIV) technique and obtained the expression for the suspension inertial force, $F_{l, m}$ :

$$
\begin{equation*}
F_{I, m}=\frac{d\left(\iiint \rho_{m} u_{m} \delta V\right)}{d t}=\zeta \frac{d}{d t}\left[\rho_{m}\left(\frac{4}{3} \pi r_{b}^{3}\right) u_{b}\right] \tag{2}
\end{equation*}
$$

where the apparent density of the suspension is defined as:

$$
\begin{equation*}
\rho_{m}=\varepsilon_{s} \rho_{s}+\varepsilon_{l} \rho_{l} . \tag{3}
\end{equation*}
$$

For bubbles formed in liquid-solid suspensions, the coefficient $\zeta$ is equal to 3.86 (Luo et al., 1998). When bubbles are formed in liquids, the coefficient $\zeta$ is equal to $\frac{11}{16}$, corresponding to the added mass in inviscid liquids (Milne-Thomson, 1955). The detailed descriptions of these forces are given in Luo et al. (1998).

In the expansion stage, the rise velocity of the bubble, $u_{b}$, is equal to the bubble expansion velocity, i.e.,

$$
\begin{equation*}
u_{b}=u_{e}=\frac{d r_{b}}{d t} \tag{4}
\end{equation*}
$$

and the gas flow rate through the orifice, $Q_{0}$, can be expressed by the following equation:

$$
\begin{equation*}
Q_{0}=\frac{d V_{b}}{d t}=4 \pi r_{b}^{2} \frac{d r_{b}}{d t} \tag{5}
\end{equation*}
$$

Substituting the expressions of various forces in Table 1 into Eq. (1) and considering Eqs. (4) and (5), the force balance at the end of the expansion stage can be written as:

$$
\begin{align*}
\frac{d}{d t}\left[\left(\rho_{g}+\zeta \rho_{m}\right)\left(\frac{4}{3} \pi r_{b}^{3}\right) \frac{d r_{b}}{d t}\right] & =\frac{4 \pi r_{b}^{3}}{3}\left(\rho_{l}-\rho_{g}\right) g+\frac{\rho_{g} Q_{0}^{2}}{\frac{1}{4} \pi D_{0}^{2}}-6 \pi \mu_{l} r_{b} \frac{d r_{b}}{d t}  \tag{6}\\
& -\pi D_{0} \sigma \cos \gamma-\frac{1}{4} \pi D_{0}^{2} \rho_{s} \varepsilon_{s}\left(\frac{d r_{b}}{d t}\right)^{2}
\end{align*}
$$

The term on the left-hand side represents the inertial forces of the bubble and suspension. The five terms on the right-hand side represent the buoyancy, gas momentum, viscosity, surface tension and particle-bubble collision forces, respectively.

In the detachment stage, the rise velocity of the bubble is the sum of the expansion velocity $\left(u_{e}\right)$ and the bubble base rising velocity ( $u$ ):

$$
\begin{equation*}
u_{b}=u+u_{e}=\frac{d x}{d t} \tag{7}
\end{equation*}
$$

where $x$ is the vertical distance between the bubble center and orifice plate. The gas flow rate through the orifice can be expressed by:

$$
\begin{equation*}
Q_{0}=\frac{d\left(V_{b}+V_{\text {neck }}\right)}{d t}=\frac{d\left[\frac{4}{3} \pi r_{b}^{3}+\frac{1}{4} \pi D_{0}^{2}\left(x-r_{b}\right)\right]}{d t}=4 \pi r_{b}^{2} \frac{d r_{b}}{d t}+\frac{1}{4} \pi D_{0}^{2}\left(\frac{d x}{d t}-\frac{d r_{b}}{d t}\right) \tag{8}
\end{equation*}
$$

Then, the motion equation of a bubble in the detachment stage can be written as follows:

$$
\begin{array}{r}
\frac{d}{d t}\left[\left(\rho_{g}+\zeta \rho_{m}\right)\left(\frac{4}{3} \pi r_{b}^{3}\right) \frac{d x}{d t}\right]=\frac{4 \pi r_{b}^{3}}{3}\left(\rho_{l}-\rho_{g}\right) g+\frac{\rho_{g} Q_{0}^{2}}{\frac{1}{4} \pi D_{0}^{2}}-6 \pi \mu_{l} r_{b} \frac{d x}{d t}-\pi D_{0} \sigma \cos \gamma  \tag{9}\\
\\
-\pi r_{b}^{2} \rho_{s} \varepsilon_{s}\left(\frac{d\left(x-r_{b}\right)}{d t}\right)^{2}-12 r_{b}^{2} \sqrt{\pi \rho_{l} \mu_{l} t} \frac{d^{2}\left(x-r_{b}\right)}{d t^{2}}
\end{array}
$$

The terms on the right-hand side represent the buoyancy, gas momentum, viscosity, surface tension, particle-bubble collision and Basset forces, respectively.

When bubbles are formed under constant flow conditions, only the motion equation of bubble is needed to simulate the bubble formation process. However, when bubbles are formed under variable flow conditions, i.e., a large gas chamber underneath the orifice, the gas flow rate through the orifice varies and depends on the pressure difference between the gas chamber and bubble. In order to simulate bubble formation under such conditions, the additional model equations, such as orifice equation, pressure balance equation at bubble-liquid interface and thermodynamic equation in the gas chamber, are required to account for the pressures inside the bubble and gas chamber. These equations are summarized next.

## Orifice Equation

The instantaneous gas flow rate through the orifice depends on the pressure difference between the gas chamber and bubble, and the orifice resistance. The following orifice equation is applicable (Kupferberg and Jameson, 1969):

$$
\begin{equation*}
\left|P_{c}-P_{b}\right|=\left(\frac{Q_{0}}{k_{0}}\right)^{2} \tag{10}
\end{equation*}
$$

$Q_{0}$ is related to the rate of bubble volume change and can be calculated by using Eqs. (5) through (8) for the expansion stage and detachment stage, respectively. The orifice constant, $k_{0}$, is a function of gas flow rate $Q_{0}$, gas density $\rho_{g}$, gas viscosity $\mu_{g}$, orifice diameter $D_{0}$, and orifice plate thickness $L$. Considering the pressure drop due to the sudden enlargement and contraction at the orifice and the frictional head loss, the following equation can be used to calculate $k_{0}$ (McAllister et al., 1958):

$$
\begin{equation*}
k_{0}=\sqrt{\frac{2}{\rho_{g} C_{g}}}\left(\frac{\pi D_{0}^{2}}{4}\right) \tag{11}
\end{equation*}
$$

where

$$
\begin{equation*}
C_{g}=1.5+4 f \frac{L}{D_{0}} . \tag{12}
\end{equation*}
$$

Assuming that the flow of gas through the orifice is laminar, the friction factor, $f$, can be expressed by

$$
\begin{equation*}
f=\frac{16}{R e_{0}}=\frac{4 \pi \mu_{g} D_{0}}{\rho_{g} Q_{0}} \tag{13}
\end{equation*}
$$

and substituting Eq. (13) into Eq. (12), the constant $C_{g}$ can be estimated by the following equation:

$$
\begin{equation*}
C_{g}=1.5+\frac{16 \pi \mu_{g} L}{\rho_{g} Q_{0}} \tag{14}
\end{equation*}
$$

Therefore, the pressure difference between the bubble and gas chamber can be calculated based on Eqs. (10), (11) and (14), if the gas flow rate through the orifice, properties of the gas phase and orifice geometry are known.

## Pressure Balance at Bubble-Liquid Interface

The pressure inside the bubble can be obtained by considering the pressure balance at the bubble-liquid interface. The change of pressure along the bubble-liquid interface is mainly due to the inertia, surface tension and viscosity of the liquid, and to the gas momentum, which can be expressed by the modified Rayleigh equation (Pinczewski, 1981):

$$
\begin{equation*}
P_{b}-P_{0}=\Delta P_{I}+\Delta P_{\mu}+\Delta P_{\sigma}-\Delta P_{m} . \tag{15}
\end{equation*}
$$

The average hydrostatic pressure at the bubble surface, $P_{0}$, is approximately expressed by the hydrostatic pressure at the orifice plate as

$$
\begin{equation*}
P_{0}=P_{s}+\rho_{l} g h \tag{16}
\end{equation*}
$$

where $P_{s}$ is the system pressure and $h$ is the liquid level in the column. The four terms on the right-hand side of Eq. (15) represent the contributions of liquid inertial, viscosity, surface tension, and gas momentum forces, respectively.

For inviscid liquids, the inertial term in Eq. (15) has been expressed in the literature based on the velocity potential around an expanding, rising bubble by Kotake, 1966; Witze et al., 1968; Kupferberg and Jameson, 1969; McCann and Prince, 1969; and Haynes and Gotham, 1982:

$$
\begin{equation*}
\Delta P_{I}=\rho_{l}\left(r_{b} \frac{d^{2} r_{b}}{d t^{2}}+\frac{3}{2}\left(\frac{d r_{b}}{d t}\right)^{2}-g x-\frac{1}{4}\left(\frac{d x}{d t}\right)^{2}\right) \tag{17}
\end{equation*}
$$

However, Eq. (17) is valid only for ideal liquids. For highly viscous liquids or liquidsolid suspensions, the flow field around the bubble is different from that for inviscid liquids, and hence the difference in the inertial term. Marmur and Rubin (1976) used the "added mass" concept and the velocity of the interface to define the inertia of the interface. They found that the "added mass" coefficient for the bubble formation process is higher than $\frac{11}{16}$, the theoretical value for a sphere moving in the vicinity of a wall. The reason is that in the bubble formation process, an expansion motion is superimposed on
the translatory motion of the bubble, and the flow field around the expanding and rising bubble is different from that for the theoretical value. In their study, they found that the value of the "added mass" coefficient is 0.85 , which agrees well with their experimental results. Luo et al. (1998) used the PIV technique to measure the flow field of liquid-solid suspensions around an accelerating bubble. They found that the flow field in liquid-solid suspensions is quite different from that in liquids, resulting in an increase in inertia.
Therefore, in order to use Eq. (15) to simulate the bubble formation in liquid-solid suspensions, the effect of particles on the inertial term must be considered. The pressure change due to the suspension inertia can be expressed by

$$
\begin{equation*}
\Delta P_{I}=\frac{F_{I, m}}{A_{b}} \tag{18}
\end{equation*}
$$

where $A_{b}$ is the surface area of the bubble and $F_{I, m}$ is the suspension inertial force expressed by Eq. (2). Substituting Eq. (2) into Eq. (18) and considering the different expressions of the bubble velocity in the two stages, the inertial term in Eq. (15) can be modified to

$$
\begin{array}{ll}
\Delta P_{I}=\zeta \rho_{m}\left[\frac{r_{b}}{3} \frac{d^{2} r_{b}}{d t^{2}}+\left(\frac{d r_{b}}{d t}\right)^{2}\right] & \text { (expansion stage) } \\
\Delta P_{I}=\zeta \rho_{m}\left[\frac{r_{b}}{3} \frac{d^{2} x}{d t^{2}}+\frac{d r_{b}}{d t} \frac{d x}{d t}\right] & \text { (detachment stage). } \tag{19b}
\end{array}
$$

For a real liquid, it is difficult to obtain theoretically a precise expression of the pressure change across the bubble-liquid interface. The common approach used is to include the terms accounting for the effects of viscosity, surface tension and gas momentum on pressure change in the Rayleigh equation, as shown in Eq. (15). Poritsky (1952) and Miyahara and Takahashi (1984) considered the effect of viscosity from the stress conditions on the bubble surface and derived the expression for the viscous term in the modified Rayleigh equation:

$$
\begin{equation*}
\Delta P_{\mu}=\frac{4 \mu_{l}}{r_{b}} \frac{d r_{b}}{d t} . \tag{20}
\end{equation*}
$$

The change in pressure along the bubble-liquid interface due to the surface tension is given by (Levich, 1962):

$$
\begin{equation*}
\Delta P_{\sigma}=\frac{2 \sigma}{r_{b}} . \tag{21}
\end{equation*}
$$

Pinczewski (1981) derived an expression for pressure distribution at the interface due to the gas momentum by assuming that the gas inside the growing bubble follows a circulatory, toroidal motion:

$$
\begin{equation*}
\Delta P_{m, \theta}=\frac{1}{2} \rho_{g} u_{0}^{2} \cos \theta \tag{22}
\end{equation*}
$$

where $\theta$ is the angle between the radial and vertical directions at any position of the bubble-liquid interface. Integrating Eq. (22) along the bubble surface, the average pressure change due to the gas momentum can be obtained:

$$
\begin{equation*}
\Delta P_{m}=\frac{\int_{0}^{2 \pi} \Delta P_{m, \theta} d \theta}{2 \pi}=\frac{1}{4} \rho_{g} u_{0}^{2} \tag{23}
\end{equation*}
$$

Substituting Eqs. (19) through (21) and (23) into Eq. (15), the modified Rayleigh equation can be extended for bubble formation in liquid-solid suspensions.

For the expansion stage:

$$
\begin{equation*}
P_{b}-P_{0}=\zeta \rho_{m}\left[\frac{r_{b}}{3} \frac{d^{2} r_{b}}{d t^{2}}+\left(\frac{d r_{b}}{d t}\right)^{2}\right]+\frac{2 \sigma}{r_{b}}+\frac{4 \mu_{l}}{r_{b}} \frac{d r_{b}}{d t}-\frac{1}{4} \rho_{g}\left(\frac{Q_{0}}{\frac{1}{4} \pi D_{0}^{2}}\right)^{2} \tag{24a}
\end{equation*}
$$

For the detachment stage:

$$
\begin{equation*}
P_{b}-P_{o}=\zeta \rho_{m}\left[\frac{r_{b}}{3} \frac{d^{2} x}{d t^{2}}+\frac{d r_{b}}{d t} \frac{d x}{d t}\right]+\frac{2 \sigma}{r_{b}}+\frac{4 \mu_{l}}{r_{b}} \frac{d r_{b}}{d t}-\frac{1}{4} \rho_{g}\left(\frac{Q_{0}}{\frac{1}{4} \pi D_{0}^{2}}\right)^{2} . \tag{24b}
\end{equation*}
$$

The instantaneous gas flow rate through the orifice, $Q_{0}$, can be calculated by using Eqs. (5) and (8) for the expansion and detachment stages, respectively. Equation (24) expresses the change of pressure inside the bubble during the bubble formation process in liquid-solid suspensions.

## Pressure Change in the Gas Chamber

The change of pressure in the gas chamber, $P_{c}$, during the expansion and detachment stages is obtained by assuming an adiabatic or isothermal expansion process of ideal gas and applying the first law of thermodynamics to the gas in the chamber (Wilkinson and van Dierendonck, 1994):

$$
\begin{equation*}
\frac{d P_{c}}{d t}=\frac{\gamma}{V_{c}}\left(P_{e} Q_{g}-P_{c} Q_{0}\right) \tag{25}
\end{equation*}
$$

where $P_{e}$ is the pressure at the gas inlet to the chamber. For the adiabatic change, $\gamma$ is the specific heat ratio of the gas $\left(\gamma=\frac{C_{p}}{C_{v}}\right)$; for the isothermal change, $\gamma$ is equal to 1 .

The radius of the bubble at the end of the expansion stage can be obtained by solving Eqs. (10), (24a) and (25) simultaneously under the following initial conditions:

$$
\begin{equation*}
t=0, r_{b}=\frac{1}{2} D_{0}, \frac{d r_{b}}{d t}=0, P_{c}=P_{s}+\rho_{m} g h+\frac{4 \sigma}{D_{0}} . \tag{26}
\end{equation*}
$$

It is assumed that the bubble is initially a hemisphere having a radius equivalent to the orifice radius. The termination of the expansion stage occurs when various forces are in balance, as given by Eq. (6). The governing equations for the detachment stage, Eqs. (9), (10), (24b) and (25), can be solved by using the final values of the expansion stage as their initial conditions:

$$
\begin{equation*}
t=t_{e}, r_{b}=r_{e}, \frac{d r_{b}}{d t}=\left.\frac{d r_{b}}{d t}\right|_{t=t_{e}}, x=r_{e}, \frac{d x}{d t}=\left.\frac{d r_{b}}{d t}\right|_{t=t_{e}}, P_{c}=P_{c, e} \tag{27}
\end{equation*}
$$

where subscript $e$ represents the end of the expansion stage. When the neck length reaches $r_{e}$, i.e., $x-r_{b}=r_{e}$, the bubble detaches from the orifice and the calculation is terminated (Ramakrishnan et al., 1969). The bubble volume at this instant gives rise to the final size of the bubble. The governing equations for both stages are coupled, ordinary differential equations and can be solved simultaneously by using the 4th-order Runge-Kutta method.

## Simulation Results

## Pressure Effect

The effect of pressure on the initial bubble size predicted by the model is shown in Figure 1 ; the solid lines represent the model predictions. It was found that the proposed model can reasonably predict the experimental results obtained in this study. The comparison between the model predictions and the experimental data in this study is provided in Figure 2, which shows that the deviation of the predictions is within $\pm 20 \%$. The pressure effect on the initial bubble size is strongly affected by the bubble formation conditions. Under variable flow conditions ( $N_{c}>1$ ), an increase in pressure significantly reduces the initial bubble size in both the liquid and the liquid-solid suspension. For bubble formation under constant flow conditions ( $N_{c} \leq 1$ ), the pressure effect is not significant.

Generally, the initial bubble size is determined by the total bubble growth time and bubble growth rate, i.e., the instantaneous gas flow rate entering the bubble $Q_{0}$. In order to analyze the pressure effect on initial bubble size under various formation conditions, it
is necessary to examine pressure effects on the bubble formation time and the gas flow rate entering the bubble. The bubble formation time and instantaneous gas flow rate through the orifice can be estimated by the proposed model. The effect of pressure on bubble formation time in a slurry $\left(\varepsilon_{s}=0.18\right)$ is shown in Figure 3. Under variable flow conditions, the bubble formation time is almost independent of the gas velocity and decreases significantly with increasing system pressure; while under constant flow conditions, the bubble formation time only slightly decreases with increasing gas velocity and system pressure. Figure 4 shows the change of gas flow rate through the orifice with time during the bubble formation process. It is seen that at ambient or low pressures, the gas flow rate through the orifice, $Q_{0}$, varies significantly with time. At the beginning of bubble growth, $Q_{0}$ increases rapidly to a maximum, which could be several times higher than the gas flow rate entering the chamber, $Q_{g}$. Then, $Q_{0}$ decreases gradually and approaches a constant value. At high pressures, $Q_{0}$ increases first to a level slightly higher than $Q_{g}$; it then decreases and quickly reaches a constant flow rate, indicating that the bubble formation is under constant flow conditions. Commonly, when $N_{c}$ is smaller than 1, the bubble formation is considered to be under constant flow conditions; however, Figure 4 shows that even under such conditions, the fluctuation in gas flow rate still exists at the beginning of bubble growth, due to the existence of a large gas chamber underneath the orifice. Figure 4 also shows that the curve of $Q_{0}$ versus time shifts to the left when the pressure increases. This indicates that the bubble starts growing earlier under high pressures. Based on the above analysis, for bubble formation under variable flow conditions, reductions in both bubble formation time and gas flow rate through the orifice with increasing pressure result in a significant decrease in initial bubble size.

The mechanism of pressure effect on bubble formation under variable flow conditions can be explained based on the proposed model. The model shows that pressure can influence the bubble formation process in the following ways: (1) the pressure directly influences the pressure fluctuation in the gas chamber based on the thermodynamic equation; (2) the pressure affects the gas momentum terms in the bubble motion equation and modified Rayleigh's equation through the change of gas density; (3) based on the orifice equation, the pressure can also influence the orifice resistance through the change in gas density, and hence the gas flow rate through the orifice. The model calculation shows that the pressure effect on the orifice constant mainly changes the gas flow rate entering the bubble. With increasing system pressure, the orifice constant becomes smaller due to the increase in gas density, and hence a decrease in the gas flow rate through the orifice. The bubble formation time is determined by the force balance on the bubble and the pressure change in the gas chamber. The increase in pressure results in a higher gas momentum rate and larger pressure fluctuation in the gas chamber, which promote the detachment of the bubble and hence decrease the bubble formation time. The combination of the above effects results in a decrease in bubble formation time and $Q_{0}$ with increasing pressure, and hence a decrease in initial bubble size.

Under constant flow conditions, the constant gas flow rate through the orifice and the weak dependence of bubble formation time on pressure contribute to an insignificant effect of pressure on initial bubble size (Figure 3). When the bubble is formed under constant flow conditions, the bubble formation process would only be influenced by
pressure through the gas momentum, as illustrated by the bubble motion equation. At low gas velocities, the influence of gas momentum would be negligible, provided the pressure or gas density is low, resulting in an insignificant change in bubble formation time with pressure, as shown in Figure 3. When the gas velocity is high, the influence of gas momentum becomes large. The bubble tends to detach from the orifice earlier due to an increased gas momentum with the increase in gas density, which results in a reduction in bubble formation time with increasing pressure and gas velocity.

## Particle Effect

Figure 5 shows the particle effect on the initial bubble size under various bubble formation conditions. Under both constant flow and variable flow conditions, the bubbles formed in the liquid-solid suspension are larger than those formed in the liquid at a given gas velocity for the present system. For bubble formation in the liquid-solid suspension, a further increase in solids concentration only slightly increases bubble size. The proposed model can also predict the trend of the particle effect. Based on the model calculations, the presence of particles in the liquid mainly influences the bubble growth time due to various forces induced by the particles on the bubble surface, such as the suspension inertial force. The particles yield resistant forces for the detachment of bubbles, resulting in longer bubble formation time and hence larger bubble size.

## Prediction of Literature Data

To examine the applicability of the proposed model, the model was used to predict the available experimental data in the literature on bubble formation under high-pressure or high gas density conditions. Figure 6 compares the model predictions and the experimental data in the literature, and shows that the proposed model can reasonably predict these data. The average error of the predictions is $18 \%$. Relevant information from various references used in Figure 6 is summarized in Table 2.

## Bubble Growth and Pressure Fluctuation in the Gas Chamber

Variations in bubble volume and pressure in the gas chamber with time simulated by the model are plotted in Figure 7. The effect of pressure on the bubble growth curve is shown in Figure 7(a). In the pressure range of 0.1 to 2.5 MPa , bubble formation is under variable flow conditions, and the bubble growth rate varies during the formation process. Under such conditions, with an increase in pressure, the bubble growth rate decreases and the bubble growth time becomes shorter. When the pressure is higher than 2.5 MPa , the bubble growth curves are nearly straight, which indicates that bubble formation is under constant flow conditions. Under such conditions, an increase in pressure does not change the bubble growth rate significantly; however, it reduces the bubble growth time.

A typical variation in pressure in the gas chamber with bubble formation time is shown in Figure 7(b). At the beginning of bubble formation, the pressure in the gas chamber increases first due to the inflow of gas to the chamber and the inertia of liquids. Because of this increase in pressure, the bubble begins to form. When the bubble growth rate is higher than the gas rate supplied to the chamber, the amount of gas and hence the pressure in the chamber begins to decrease. As shown in Figure 7(b), pressure has a significant effect on pressure fluctuation in the gas chamber. At ambient pressure, the
pressure fluctuation in the chamber is relatively small. With an increase in pressure, the pressure fluctuation in the chamber becomes significant. At high pressures, it is also seen that the pressure in the gas chamber becomes constant after a certain time. For example, at a pressure of 2.5 MPa , after 20 ms the pressure in the gas chamber becomes constant, as shown in Figure 7(b). This can be illustrated by considering the change of $Q_{0}$ with the growth time. As shown in Figure 4, when the pressure is 2.5 MPa , after 20 ms , the gas flow rate to the bubble is constant and approximately equal to the gas flow rate entering the gas chamber. This results in a constant amount of gas in the chamber, and hence a constant pressure. As shown in Figures 4 and 7, it is clear that even when bubble formation is under constant flow conditions, there still exist fluctuations in pressure in the gas chamber and in the gas flow rate through the orifice at the beginning of bubble formation.

## Notations

$A_{b} \quad$ surface area of bubble, $\mathrm{m}^{2}$
$C_{D} \quad$ drag coefficient, dimensionless
$C_{g} \quad$ constant in Eq. (11), dimensionless
$C_{p} \quad$ specific heat capacity of gas at constant pressure, $\mathrm{J} /(\mathrm{kg} \cdot \mathrm{K})$
$C_{v} \quad$ specific heat capacity of gas at constant volume, $\mathrm{J} /(\mathrm{kg} \cdot \mathrm{K})$
$D_{0} \quad$ orifice diameter, m
$d_{b} \quad$ initial bubble size, m
$e \quad$ restitution coefficient, dimensionless
$f$ friction factor, dimensionless
$F_{\sigma} \quad$ surface tension force, N
$F_{B} \quad$ buoyancy force, N
$F_{\text {Basset }}$ Basset force, N
$F_{C} \quad$ particle-bubble collision force, N
$F_{D} \quad$ liquid drag force, N
$F_{I, g} \quad$ bubble inertial force, N
$F_{l, m} \quad$ suspension inertial force, N
$F_{M} \quad$ gas momentum force, N
$g \quad$ gravitational acceleration, $\mathrm{m} / \mathrm{s}^{2}$
$h \quad$ liquid level in the column, $m$
$k_{0} \quad$ orifice constant, $\mathrm{m}^{3} /\left(\mathrm{Pa}^{0.5} \cdot \mathrm{~s}\right)$
$L \quad$ thickness of orifice plate, $m$
$N_{c} \quad$ capacitance number, dimensionless
$P_{0} \quad$ hydrostatic pressure at the bubble surface, Pa
$P_{b} \quad$ pressure in the bubble, Pa
$P_{c} \quad$ pressure in the gas chamber, Pa
$P_{c, e} \quad$ pressure in the gas chamber at the end of expansion stage, Pa
$P_{e} \quad$ pressure at the gas inlet to the chamber, Pa
$P_{e} \quad$ pressure at the gas inlet to the chamber, Pa
$P_{s} \quad$ system pressure, Pa
$Q_{0} \quad$ instantaneous gas flow rate through the orifice, $\mathrm{m}^{3} / \mathrm{s}$
$Q_{g} \quad$ volumetric gas flow rate entering the gas chamber, $\mathrm{m}^{3} / \mathrm{s}$
$r_{b} \quad$ bubble radius, $m$
$r_{e} \quad$ bubble radius at the end of expansion stage, $m$
$\operatorname{Re} \quad$ Reynolds number, $\frac{2 \rho_{l} u_{b} r_{b}}{\mu_{l}}$, dimensionless
$R e_{o} \quad$ orifice Reynolds number, $\frac{\rho_{g} u_{0} D_{0}}{\mu_{g}}$, dimensionless
$T$ temperature, K
$t$ time, s
$t_{e} \quad$ time of expansion stage, s
$u \quad$ rise velocity of bubble base, $\mathrm{m} / \mathrm{s}$
$u_{0} \quad$ instantaneous gas velocity through the orifice, $\mathrm{m} / \mathrm{s}$
$u_{b} \quad$ rise velocity of bubble center, $\mathrm{m} / \mathrm{s}$
$u_{e} \quad$ bubble expansion velocity, $\mathrm{m} / \mathrm{s}$
$u_{g} \quad$ orifice gas velocity, $\mathrm{m} / \mathrm{s}$
$u_{m} \quad$ velocity of liquid-solid suspensions, $\mathrm{m} / \mathrm{s}$
$V_{b} \quad$ volume of bubble, $\mathrm{m}^{3}$
$V_{c} \quad$ volume of gas chamber, $\mathrm{m}^{3}$
$V_{\text {neck }} \quad$ volume of neck, $\mathrm{m}^{3}$
$x \quad$ distance between bubble center and orifice plate, m

## Greek Letters

$\theta$ angle between the radial and vertical directions of interface element, rad
$\varepsilon_{l} \quad$ liquid holdup, dimensionless
$\varepsilon_{s} \quad$ solids concentration, dimensionless
$\gamma \quad$ contact angle between bubble surface and orifice, rad specific heat ratio of gas, dimensionless
$\mu_{g} \quad$ gas viscosity, $\mathrm{Pa} \cdot \mathrm{s}$
$\mu_{l} \quad$ liquid viscosity, $\mathrm{Pa} \cdot \mathrm{s}$
$\rho_{g} \quad$ gas density, $\mathrm{kg} / \mathrm{m}^{3}$
$\rho_{l} \quad$ liquid density, $\mathrm{kg} / \mathrm{m}^{3}$
$\rho_{m} \quad$ density of liquid-solid suspensions, $\mathrm{kg} / \mathrm{m}^{3}$
$\rho_{s} \quad$ particle density, $\mathrm{kg} / \mathrm{m}^{3}$
$\sigma$ surface tension, $\mathrm{N} / \mathrm{m}$
$\zeta \quad$ coefficient in Eq. (2), dimensionless
$\Delta P_{I} \quad$ pressure change along the bubble-liquid interface due to liquid inertia, Pa
$\Delta P_{m} \quad$ pressure change along the bubble-liquid interface due to gas momentum, Pa
$\Delta P_{m, \theta}$ pressure change at any interface element due to gas momentum, Pa
$\Delta P_{\mu} \quad$ pressure change along the bubble-liquid interface due to liquid viscosity, Pa
$\Delta P_{\sigma} \quad$ pressure change along the bubble-liquid interface due to surface tension, Pa

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Figure 1 Effect of Pressure on the Initial Bubble Size ( $T=\mathbf{3 0}{ }^{\circ} \mathrm{C}, \mathrm{D}_{\mathbf{0}}=\mathbf{1 . 6 3} \mathbf{~ m m}, V_{c}=\mathbf{6 5 0}$ $\mathbf{c m}^{3}$ ): (a) in the liquid ( $\varepsilon_{\mathrm{s}}=0$ ); and (b) in the liquid-solid suspension ( $\varepsilon_{\mathrm{s}}=\mathbf{0} .18$ ) (symbols: experimental data; lines: model predictions)


Figure 2 Comparison Between the Model Predictions and Experimental Data of the Initial Bubble Size under High-Pressure Conditions


Figure 3 The Bubble Formation Time as a Function of Gas Velocity at Various Pressures in the Liquid-Solid Suspension ( $\varepsilon_{\mathrm{s}}=\mathbf{0 . 1 8}$ )


Figure 4 Effect of Pressure on the Variation of Gas Flow Rate Through the Orifice with Time during the Bubble Formation Process in a Liquid-Solid Suspension

$$
\left(\varepsilon_{\mathrm{s}}=0.18, \mathbf{Q}_{\mathrm{g}}=2 \mathrm{~cm}^{3} / \mathrm{s}\right)
$$



Figure 5 Effect of Solids Concentration on Initial Bubble Size ( $\mathbf{T}=\mathbf{3 0}{ }^{\circ} \mathrm{C}, \mathrm{D}_{\mathbf{0}}=\mathbf{1 . 6 3 m m}$, $V_{c}=650 \mathrm{~cm}^{3}$ ): (a) variable flow conditions ( $P_{s}=0.3 \mathrm{MPa}, \mathrm{N}_{\mathrm{c}}=8.8$ ); and (b) constant flow conditions ( $\mathrm{P}_{\mathrm{s}}=4.9 \mathrm{MPa}, \mathrm{N}_{\mathrm{c}}=\mathbf{0 . 5}$ ) (symbols: experimental data; lines: model predictions).


- Tsuge et al. (1992)
- Tsuge et al. (1992)
\Delta Wilkinson and van Dierendonck (1994)
\Delta Wilkinson and van Dierendonck (1994)
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XKling (1962) + Yoo et al. (1998)
\Delta this work * Luo et al. (1998a)
\Delta this work * Luo et al. (1998a)

Figure 6 Comparison between the Model Predictions and the Literature Data of Initial Bubble Size under High-Pressure Conditions (For information about literature data, see Table 2.)


Figure 7 Effect of Pressure on the Variation of (a) Bubble Volume and (b) Pressure in Gas Chamber with Time during the Bubble Formation Process $\left(\varepsilon_{\mathrm{s}}=\mathbf{0 . 1 8}, \mathrm{Q}_{\mathrm{g}}=\mathbf{2}\right.$ $\mathrm{cm}^{3} / \mathrm{s}$ )

Table 1 Expressions of the Forces involved in the Bubble Formation Process (Luo et al., 1998)

| FORCES | EXPANSION STAGE | DETACHMENT STAGE |
| :---: | :---: | :---: |
| $F_{B}$ | $\frac{4 \pi}{3} r_{b}^{3}\left(\rho_{l}-\rho_{g}\right) g$ | Same as expansion stage |
| $F_{M}$ | $\frac{\pi}{4} D_{0}^{2} \rho_{g} u_{o}^{2}$ | Same as expansion stage |
| $F_{D}$ | $C_{D}\left(\pi r_{b}{ }^{2}\right) \frac{\rho_{l} u_{b}^{2}}{2} \quad\left(C_{D}=24 / \mathrm{Re}\right)$ | Same as expansion stage |
| $F_{\sigma}$ | $\frac{\pi}{d t}\left[\rho_{g}\left(\frac{4}{3} \pi r_{b}^{3}\right) u_{b}\right]$ | Same as expansion stage |
| $F_{l, g}$ | Not applicable | Same as expansion stage |
| $F_{B a s s e t}$ | $\frac{\pi}{4} D_{0}{ }^{2}(1+e) \varepsilon_{s} \rho_{s} u_{e}{ }^{2}$ | $12 r_{b}^{2} \sqrt{\pi \rho_{l} \mu_{l} t} \frac{d u}{d t}$ |
| $F_{C}$ | $\frac{\mathrm{~d}\left(\iint \rho_{\mathrm{m}} u_{m} \delta V\right)}{\mathrm{d} t}=3.86 \frac{\mathrm{~d}}{\mathrm{~d} t}\left[\rho_{m}\left(\frac{4}{3} \pi r_{b}^{3}\right) u_{b}\right]$ | $\pi r_{b}^{2} \varepsilon_{s} \rho_{s} u^{2}$ |
| $F_{l, m}$ | Same as expansion stage |  |

Table 2 Relevant Information from Various References used in Figure 6 concerning Bubble Formation under High-Pressure Conditions

| Reference | System | $\mathrm{P}_{\text {S }}(\mathrm{MPa})$ | $\mathrm{V}_{\mathrm{c}}\left(\mathrm{cm}^{3}\right)$ | $\mathrm{D}_{0}(\mathrm{~mm})$ | $\mathrm{N}_{\mathrm{c}}$ | $\mathrm{Q}_{\mathrm{g}}\left(\mathrm{cm}^{3} / \mathrm{s}\right)$ | Symbol in Fig. 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Kling (1962) | $\mathrm{N}_{2}, \mathrm{Ar} / \mathrm{H}_{2} \mathrm{O}$ | $0.3 \sim 8.1$ | 9.08 | 1.05 | $0.01 \sim 0.34$ | $0.1 \sim 5.0$ | $\times$ |
| LaNauze and Harris (1974) | $\mathrm{CO}_{2} / \mathrm{H}_{2} \mathrm{O}$ | $0.1 \sim 2.17$ | 375 | 4.8 | 0.09~2.0 | $1.0 \sim 30.0$ | $\checkmark$ |
| Bier et al. (1978) | $\mathrm{N}_{2} / \mathrm{H}_{2} \mathrm{O}$ | $0.1 \sim 2.1$ | 1775 | 1.6 | 4.1 ~ 84 | $0.1 \sim 20.0$ | $\stackrel{\text { r }}{ }$ |
| Tsuge et al. (1981) | $\begin{aligned} & \mathrm{He}, \mathrm{~N}_{2}, \mathrm{Ar}, \\ & \mathrm{CO}_{2} / \mathrm{H}_{2} \mathrm{O} \\ & \hline \end{aligned}$ | 0.1 | 153 | 1.08 | 16 | $0.1 \sim 10.0$ | - |
| Tsuge and Hibino (1983) | $\begin{aligned} & \mathrm{He}, \mathrm{~N}_{2}, \mathrm{Ar}, \\ & \mathrm{CO}_{2} / \mathrm{H}_{2} \mathrm{O} \\ & \hline \end{aligned}$ | 0.1 | 237 | 1.08 | 25 | $0.1 \sim 10.0$ |  |
| Idogawa et al. (1985) | air/water | $1.0 \sim 5.0$ | 8.1 | 1.0 | $0.2 \sim 1.0$ | 0.5~5.0 | $\bigcirc$ |
| Tsuge et al. (1992) | $\mathrm{N}_{2} / \mathrm{H}_{2} \mathrm{O}$ | $1.0 \sim 8.1$ | 365 | 1.48 | $0.3 \sim 2.1$ | $0.1 \sim 5.0$ | $\bigcirc$ |
| Wilkinson and van Dierendonck (1994) | $\mathrm{N}_{2} / \mathrm{H}_{2} \mathrm{O}$ | $0.1 \sim 2.1$ | 1775 | 1.6 | $0.4 \sim 8.7$ | $0.1 \sim 10.0$ | - |
| Yoo et al. (1997) | air/glycerol/ polystyrene beads | $0.1 \sim 8.0$ | 86.2 | 1.18 | $0.1 \sim 8.1$ | $0.1 \sim 4.0$ | $\square$ |
| Yoo et al. (1998) | $\mathrm{CO}_{2}, \mathrm{~N}_{2} / \mathrm{H}_{2} \mathrm{O}$ | $0.1 \sim 8.0$ | 86.2 | 1.48 | 0.06~4.9 | 0.1~5.0 | + |
| Luo et al. (1998) | $\mathrm{N}_{2} / \mathrm{NF}$ fluid /glass beads | $0.1 \sim 17.3$ | Constant flow | 1.59 | 1.0 | $0.002 \sim 8.0$ | * |
| This work (1999) | $\mathrm{N}_{2}$ /NF fluid /glass beads | $0.1 \sim 8.3$ | 650 | 1.63 | $0.3 \sim 26.5$ | $0.1 \sim 15.0$ | $\Delta$ |

