

# CIRCULATION IN GAS-SLURRY COLUMN REACTORS: SEVENTH QUARTERLY REPORT, QUARTER ENDING JUNE 30, 1989 

WEST VIRGINIA UNIV., MORGANTOWN. DEPT. OF MECHANICAL AND AEROSPACE ENGINEERING

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#### Abstract

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## 1. INTRODUCTION

During this quarter progress has been made in all 3 areas of the bubble coldamn research (prode measurements, laser doppler anemometry and numerical modeling of the two phase flow). In particular, bubble velocities can now be inferred from cross-correlation of two probe signals, and the numerical modeling has yielded circulation streamlines for a bubble column in laminar flow. Liquid velocities can also be measured in a hexagonal test cclumn with the laser doppler velocimeter. Details of this research progress follow.

## 2. RESISTANCE PROBE MEASUREMENTS

The bulk of the air-water void profile data gathering is now complete, with profiles available for two traverses (at right angles to one another) at varying heights of mixture in the column. Data has been taken at several different heights and different air flowrates in each column. Figures 2.1 to 2.6 provide data on the column containing an unaerated water height of 12 inches. Figures 2.7 to 2.18 present data for the unaerated water height of 24 inches and Figures 2.19 to 2.27 give an incomplete data set for the case of an unaerated water height of 36 inches. Note that the voidage distribution is initially relatively uniform versus radius near the bottom of the column, but becomes more non-uniform as the top of the column is approached. Voidage is highest on the centerline. These data indicate that the initial uniform injection of bubbles at the distributor plate is altered by an inwards migration of bubbles towards the centerline of the column as they rise up the column.

[^0]for this purpose, and some processed data are already available. Figure 2.30 shows the shear-stress profile associated with the void fraction distribution and mean local density distribution shown in Figures 2.31 and 2.32 respectively. Figure 2.33 shows the liquid velocity distribution predicted from the shear stress distribution using water viscosity and a Prandtl mixing length approach. Figure 2.34 shows three predicted liquid velocity profiles arising from actual void data zaken in the column. This will be compared with bubble velocity data acquired from the column during the next quarter.
3. LASER DOPPLER VELOCIMETRY WORK

Preliminary measurements of liquid vertical velocity in a hexagonal bubble column have been made during the past quarter using the laser doppler velocimeter (LDV). Efforts during the next quarter are aimed at obtaining simultaneous measurement of all three components of liquid velocity, and at also measuring the bubble velocity components nonintrusively using the LDV.

A schematic of the hexagonal cross-section bubble column is shown in Figure 3.1: The column is made of plexiglas, and is 18 cm across ine flats or 21 cm across the diagonal, and can accommodate water depths up to 18 cm . For the present, preliminary experiments an air bubble injection manifold with a single, central hole of $1 / 16$ inch diameter has been used. This results in an initial jet of air which breaks into nominal 1-2 cm diameter bubbles approximately one third of the distance up the column, which drives a relatively strong water circulation which is upwards near the central column of
air bubbles, and downwards near the outer walls of the column. This configuration is being studied first because it greatly simplifies the measurement of liquid velocity because there is very little interference with the laser beams due to bubbles crossing through the beam paths away from where the laser beams cross in the measurement volume to form interference fringes. Also, the hexagonal cross-section has been selected because it eliminates the prodems associated with penetrating a curved interface, while still remaining nearly cylindrical in shape. Once better alignment methods are developed, it is hoped that it will be possible to view into the flowfield normal to one face of the coiumn with the 2 -channel LDV, while simultaneously focusing on the same location with the 1 -channel LDV system through a neighboring face of the column, thereby allowing 3-D velocity measurements to be made.

Preliminary vertical liquid velocity measurements have been measured versus vertical depth at the three different radial locations indicated in Figure 3.1 (2.6, 4.5, and 6.4 cm from the centerline). Air flow rate was nominally 2 SCFH. Average vertical liquid velocity (in $\mathrm{cm} / \mathrm{sec}$ ) versus vertical coordinate (in cm ) is shown in Figure 3.2, while RMS liquid vertical velocity is shown in Figure 3.3. Measured vertical liquid velocity is positive (upwards) at 2.5 cm and 4.5 cm (near mid-radius), and essentially zero or slightly negative at 6.4 cm (approximately two-thirds the radius). Future measurements will be made at larger radii to confirm the downwards liquid velocity expected beyond 6.4 cm . Vertical liquid mean velocity is larger in the top half of the column. RMS liquid velocity is the same order of magnitude as, or larger than, the mean velocity. Fluctuations are
largest at smaller radii in the top half of the column.
For these preliminary liquid velocity measurements at an air flow rate of 2 SCFH, there is essentially no re-entrainment of the air into Ehe downward moving outer liquid flow. Thus, all LDV signals were generated by the silver-coated $5 \mathrm{\mu m}$ diameter glass microspheres which were used to seec the liquid flow. Particles of this size essentially follow the liquid flow.
it is planned that in the future liquid vertical velocities will be measured at a greater number of radii, and that the radial liquid velocity component will be measured. Air flow rate will be increased as much as is practical, and efforts will be directed at measuring both the liquid velocity distribution and the bubble velocity fistribution. Also, it is hoped that different manifolds will be used which wiil inject air bubbles over a larger portion of the column cross section. In as much as is practical, measurements will be obtained for bubble injection manifolds and flow rates which can be simulated using the numerical model.

## 4. NUMERICAL SIMULATIONS

### 4.1 Introduction

The modeling of circulation in a bubble column reactor has been conducted to develop understanding of the process of interaction between a dispersed phase (air bubbles) and continuous phase (water). A complete mathematical model involving the momentum exchange between the two phases has been developed, with all the terms related to the two phases are developed empirically or analytically. A typical experimental and theoretical study for a liquid circulation in a
gas-liauid system has been presented by Rietema and Ottengraf (1970). The numerical simulation has been performed based on their experimental work. This section is a summary of the work done to date for the numerical simulation of circulation in a bubbly reactor.
4.2 Mathematical Model

Consider an unsteady, gas-liquid flow inside a vertically situated circular reactor which is assumed to be isothermal and non-reacting. The mathematical formulation for such two-phase flow is based on the conservation of mass and momentum principle for each phase. The gas-liquid flow is assumed to be in. the bubbly flow regime which is characterized by a suspension of discrete air bubbles in a continuous liquid such as water. Let $\rho_{1}$ be the liquid macroscopic density, and $\rho_{2}$ be the gas macroscopic density such that $\rho_{1}=(1-\alpha) \rho_{\ell}, \rho_{2}=\alpha \rho_{g}$, where $\rho_{\ell}$ and $\rho_{g}$ are the microscopic densities for liquid and gas respectively, and $\alpha$ is the void fraction for the gas. Let $\underline{u}_{1}$ be the liquid velocity vector $\left(u_{1}, v_{1}, w_{1}\right)$ and $\underline{u}_{2}$ the gas velocity vector $\left(u_{2}, v_{2}, w_{2}\right)$ in the axial $x-$, radial $r-$, and tangential, $\theta$ - directions, respectively.

## Equations For Liquid Phase

The continuity equation representing the conservation of mass for the liquid phase is

$$
\begin{equation*}
\frac{D p_{1}}{D t}+\rho_{1}\left(\underline{\underline{u}} \cdot \underline{u_{1}}\right)=\frac{\partial \rho_{1}}{\partial t}+\underline{\underline{z}} \cdot\left(\rho_{1} \underline{u}_{1}\right)=0 \tag{4.1}
\end{equation*}
$$

where $D() / D t$ is the substantial derivative and $\geq$. is the divergence operator. The term z.ㅡ﹎ is zero for an incompressible flow. There are no source/sink terms in Eq.4.1 because there is no phase change due
to chenical reactions or thermal changes. For numerical treatment Eq.4.1 is reirritten in conservative form as

$$
\begin{equation*}
\frac{\partial \rho_{1}}{\partial t}+\frac{1}{r} \frac{\partial}{\partial r}\left(\rho_{1} r v_{1}\right)+\frac{1}{r} \frac{\partial}{\partial \theta}\left(\rho_{1} w_{1}\right)+\frac{\partial}{\partial x}\left(\rho_{1} u_{1}\right)=0 \tag{4.2}
\end{equation*}
$$

The momentum equations for liquid phase, in vector form (see for example Celik, 1986), are

$$
\begin{equation*}
\rho_{1} \frac{\underline{\underline{u}}_{1}}{D_{t}}=-(1-\alpha) P-v_{0} \underline{\underline{I}}+F_{12}\left(\underline{\underline{u}}_{2}-\underline{\underline{u}}_{1}\right)+\rho_{19}+\underline{f}_{c}-w_{1} \underline{e}_{x} \times \underline{\underline{u}}_{1} / r \tag{4.3}
\end{equation*}
$$

In Eq.(4.3), the term $3 . I$ is the stress tensor, $g$ is the acceleration of gravity, $P$ is the mixture pressure, $F_{12}$ is the momentum exchange function, $e_{x}$ is the unit vector in $x$ - direction, and $f_{c}$ is the additional stress vector ( $\left.0,-_{\theta \theta} / r, T_{r \theta} / r\right)$ in $x-, r-$, and $\theta-$ directions, respectively. The components of the symmetric stress tensor $\underset{\sim}{ }$ in cylindrical coordinates for Newtonian fluids are

$$
\begin{align*}
& T_{x x}=-P+2 \mu \frac{\partial u}{\partial x}-\frac{2}{3} \mu \underline{\nabla} \cdot \underline{u} \\
& T_{r r}=-P+2 \mu \frac{\partial v}{\partial r}-\frac{2}{3} \mu \underline{\nabla} \cdot \underline{u} \\
& T_{\theta \theta}=-P+2 \mu\left(\frac{1}{r} \frac{\partial w}{\partial \theta}+\frac{v}{r}\right)-\frac{2}{3} \mu \underline{z} \cdot \underline{u} \\
& T_{r x}=-_{x r}=\mu\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial r}\right)  \tag{4.4}\\
& T_{r \theta}=-_{\theta r}=\mu\left(\frac{\partial w}{\partial r}+\frac{1}{r} \frac{\partial v}{\partial \theta}-\frac{w}{r}\right) \\
& T_{x \theta}=\tau_{\theta x}=\mu\left(\frac{1}{r} \frac{\partial u}{\partial g}+\frac{\partial w}{\partial x}\right)
\end{align*}
$$

The dilatation term $\mathbf{z}$. $\underline{u}$ will be negiected in the momentum equations, since the effect of this term is minimal even for a wide range of compressible fluid flows provided that the Mach number is less than 0.30. It should be noted, however, that $\underset{\text { g.u }}{ }$ will be keft in the equation of continuity and the resulting equations would be valid for many compressible fluid flow problems unless this term becomes very large under unusual circumstances, i.e. Mach numbers iarger than 0.3.

The term Fi2 ( $\underline{u}_{2}-\underline{u}_{1}$ ) in Eq. (4.3) represents the momentum exchange for liquid-phase equations, likewise, $F_{21}\left(\underline{u}_{2}-\underline{u}_{1}\right)$ is the momentum exchange term for gas-phase equations, fence $\mathrm{F}_{12}=-\mathrm{F}_{21}$. If the dilatation term $\mathrm{J}_{-1}$ is neglected, the three components of Eq.4.3 in $\mathrm{x}-\mathrm{r}, \mathrm{r}, \mathrm{a}$, directions, respectively, can be written in conservative form as follows

## $x$ - Momentum

$$
\begin{align*}
\frac{\partial \rho_{1} u_{1}}{\partial t} & +\frac{\partial}{\partial x}\left(\rho_{1} u_{1} u_{1}\right)+\frac{1}{r} \frac{\partial}{\partial r}\left(\rho_{q_{q}} u_{1}\right)+\frac{1}{r} \frac{\partial}{\partial \theta}\left(\rho_{1} u_{q} w_{1}\right) \\
& =\frac{\partial}{\partial x}\left(\tau_{x x}\right)+\frac{1}{r} \frac{\partial}{\partial r}\left(r r_{r x}\right)+\frac{1}{r} \frac{\partial}{\partial \theta}\left(r_{\theta x}\right)+\rho_{1} g_{x}+F_{12}\left(u_{2}-u_{f}\right) \tag{4.5}
\end{align*}
$$

r-Momentum

$$
\begin{align*}
& \frac{\partial \rho_{1} v_{1}}{\partial t}+\frac{\partial}{\partial x}\left(\rho_{1} u_{1} v_{1}\right)+\frac{1}{r} \frac{\partial}{\partial r}\left(\rho_{\rho_{1}} v_{1} v_{1}\right)+\frac{1}{r} \frac{\partial}{\partial \theta}\left(\rho_{1} v_{1} w_{1}\right)-\rho_{1} \frac{w_{1}}{r} \\
& =\frac{\partial}{\partial x}\left(T_{x r}\right)+\frac{1}{r} \frac{\partial}{\partial r}\left(r r_{r r}\right)+\frac{1}{r} \frac{\partial}{\partial \theta}\left(T_{\theta r}\right)-\frac{\sigma_{\theta \theta}}{r}+\rho_{1} g_{r}+F_{12}\left(v_{2}-v_{1}\right) \tag{4.6}
\end{align*}
$$

## - -Momentum

$$
\frac{\partial \rho_{1} w_{1}}{\partial t}+\frac{\partial}{\partial x}\left(\rho_{1} u_{1} w_{1}\right)+\frac{1}{r} \frac{d}{\partial r}\left(\rho_{\rho_{1}} v_{1} W_{1}\right)+\frac{1}{r} \frac{\partial}{\partial \theta}\left(\rho_{1} w_{1} w_{1}\right)+\frac{\rho_{1} v_{1} w_{1}}{r}
$$

$$
\begin{equation*}
=\frac{\partial}{\partial x}\left(\Gamma_{x_{\theta}}\right)+\frac{1}{r} \frac{\partial}{\partial r}\left(\Gamma_{\theta r}\right)+\frac{1}{r} \frac{\partial}{\partial \theta}\left(\Gamma_{\theta \theta}\right)+\frac{\bar{\mu}_{\theta}}{r}+\rho_{1 g_{\theta}}+F_{12}\left(w_{2}-w_{1}\right) \tag{4.7}
\end{equation*}
$$

For the axisymmetric, non-swirling flow case that will be investigated, Eqs.4.5-4.7 are greatly simplified. The final form of the liquid phase equations which will be solved numerically are:

Continuity

$$
\begin{equation*}
\frac{\partial \rho 1}{\partial t}+\frac{1}{r} \frac{\partial}{\partial r}\left(\rho_{1} r v_{1}\right)+\frac{\partial}{\partial x}\left(\rho_{1} u_{1}\right)=0 \tag{4.8}
\end{equation*}
$$

$x$ - Momentum

$$
\begin{align*}
\frac{\partial}{\partial t}\left(\rho_{1} u_{1}\right)+ & \frac{\partial}{\partial x}\left(\rho_{1} u_{1} u_{1}\right)+\frac{1}{r} \frac{\partial}{\partial r}\left(r \rho_{1} u_{1} v_{1}\right)=-(1-a) \frac{\partial P}{\partial x}+2 \frac{\partial}{\partial x}\left(\mu \varepsilon \frac{\partial u_{1}}{\partial x}\right) \\
& +\frac{1}{r} \frac{\partial}{\partial r}\left(\mu_{\ell} \frac{\partial u_{1}}{\partial r}\right)+\frac{1}{r} \frac{\partial}{\partial r}\left(r \mu_{\ell} \frac{\partial v_{1}}{\partial x}\right)-\rho_{1} g+F_{12}\left(u_{2}-u_{1}\right) \tag{4.9}
\end{align*}
$$

## $r$ - Momentum

$$
\begin{align*}
\frac{\partial}{\partial t}\left(\rho_{1} v_{1}\right)+ & \frac{\partial}{\partial x}\left(\rho_{1} u_{1} v_{1}\right)+\frac{1}{r} \frac{\partial}{\partial r}\left(r \rho_{1} v_{1} v_{1}\right)=-(1-\alpha) \frac{\partial P}{\partial r}+\frac{\partial}{\partial x}\left(\mu_{\ell} \frac{\partial v_{1}}{\partial x}\right) \\
& +\frac{2}{r} \frac{\partial}{\partial r}\left(r \mu_{\ell} \frac{\partial v_{1}}{\partial r}\right)+\frac{\partial}{\partial x}\left(\mu_{\ell} \frac{\partial u_{1}}{\partial r}\right)-\mu_{\ell} \frac{2 v_{1}}{r^{2}}+F_{12}\left(v_{2}-v_{1}\right) \tag{4.10}
\end{align*}
$$

Similar equations apply to the gas-phase flow. The momentum exchange functiorrf $F_{12}$ will be prescribed empirically in the functional form $F_{12}=F_{12}\left(a, R_{b}\right)$
where $\mathrm{Re}_{b}$ is the bubble Reynolds number defined as
$R e_{b}=\rho_{\ell} l \underline{u}_{2}-\underline{u}_{1} l d_{b} / \mu_{\ell}$, with $d_{b}$ being the bubble diameter. The explicit form of Eg.4.11 is discussed in the next subsection.

It should be noted that the way the pressure gradient terms should be handled in Egs. $4.8-4.10$ is a controversial issue. There is considerable debate in the literature (see for example Stewart et al., 1984) Whether $\nabla[(1-\alpha) P]$ or $(1-\alpha) \nabla P$ should be used in these equations. Both forms satisfy the condition that when the corresponding momentum equations for the two phases are added, the resulting pressure gradient term must be $\nabla$ P. The equal pressure model (Stewart et al. 1984) will be adopted, i.e. $P_{1}=P_{2}=P$; this pressure will be distributed as ( $1-\alpha$ ) $P$ and $a P$ between the liquid and gas phases, respectively. In this regard, the pressure gradient terms are being treated as part of the interfacial momentum exchange. The surface tension effects will be included in the interfacial momentum exchange function $F_{12}$; for different surface tension, $\sigma$, the drag force and hence the terminal velocity of a bubble are different. It should be noted, however, if we include a pressure difference $a p=P_{1}-P_{2}=4 \sigma / d_{b}$ due to surface tension, this does not alter the form of the above equations. This is because the bubble diameter $d_{b}$ and the strface tension, $\sigma$, are fixed for a given flow regime and hence $P_{2}=P_{2}+4 \sigma / d_{b}=P+4 \sigma / d_{b}$ and $\nabla P_{2}=\nabla P$ again.

Once $F_{12}=F_{12}\left(\alpha, R_{b}\right)$ is prescribed, Eqs.4.8 through 4.10 written for both phases constitutes a closed set of 6 differential equations for the 6 unknowns, namely $a, P, u_{1}, v_{1}, u_{2}$ and $v_{2}$. For the initial part of the numerical simulations these equations witi be reduced to 4 equations and 4 unknowns by assuming a slip velocity relation of the form

$$
\begin{equation*}
\underline{u}_{s}=\underline{u}_{2}-\underline{u}_{1}=f\left(\alpha, R_{e_{b}}\right) \tag{4.12}
\end{equation*}
$$

This explicit form of Eq.4.12 is discussed in the following sections.

## Interfacia! Momentum Exchange

The momentum transfer Detween the different phases takes place via several mechanisms, the most important of which being the viscous drag force resulting from the shear stress at the interface and the form drag due to the pressure distribution on che surface of individual bubbles. Other possible mechanisms for momentum transfer include added mass effect, magnus effect (due to rotation), pressure gradient, and shear rate effects of the surrounding fluid (see for example Hinze, 1972). For brevity these forces will not be considered in the present analysis. Instead, ali these effects will be lumped into the drag function (Eq. 4.11 .

In the bubbly flow regime, the total drag force can be related to that of a single bubble. Hirt (1982) used the following relation for water droplets in steam.

$$
\begin{equation*}
F_{12}=\frac{3}{4} \alpha^{2}(1-\alpha) \rho_{2} \frac{\underline{u}_{1}-\underline{u}_{2}^{1}}{d_{p}} C_{D} \tag{4.13}
\end{equation*}
$$

Where $d_{p}$ is the droplet particle diameter. Syamlal and O'Brien (1988) suggested the following empirical relation for dispersed solid particles in a continous liquid or gas phase

$$
\begin{equation*}
F_{12}=\frac{3}{4} \frac{\alpha^{2}(1-\alpha) \rho_{\ell} \underline{\underline{u}}_{1}-\underline{u}_{2}^{\prime}}{d_{D}} C_{D}^{*} \tag{4.14}
\end{equation*}
$$

where $C_{D}{ }^{*}$ is another empirical drag function given by

$$
\begin{equation*}
C_{D}^{*}=\frac{C_{D}\left(R_{e b}<=R_{e b} / V_{r}\right)}{V_{r}^{2}} \tag{4.15}
\end{equation*}
$$

In Eg. 4.15 $C_{D}=C_{D}\left(R_{b}\right)$ is the drag coefficient for an isolated particle and $V_{r}$ is the ratio of terminal velocity of a group of particles to that of an isolated particle. Neither of the equations 4.13 or 4.15 is strictly applicable to the bubbly flow regime. However, to a first approximation the simpler relation used by Hirt (1982) should be sufficient for our purposes, provided that $C_{D}$ is replaced by an empirical relation for bubbles in water.

Such a $C_{D}$ relation can be derived by curve fitting to the experimental data presented by clift et al. (1978). For bubbles in pure systems, the following function is suggested.

$$
\begin{equation*}
C_{D}=a \operatorname{Re}_{b}^{-b} \tag{4.16}
\end{equation*}
$$

with

| $\operatorname{Re}_{b}<2$ | $a=24$ | $b=1.000$ |
| :--- | :--- | :--- |
| $2 \leqslant \operatorname{Re}_{b}<10$ | $a=23.66$ | $b=0.981$ |
| $10 \& \operatorname{Re}_{b}<100$ | $a=14.9$ | $b=0.780$ |
| $100 \leqslant \operatorname{Re}_{b}<1000$ | $a=6.9$ | $b=0.613$ |

## Equations for Gas Phase

Instead of solving for the gas or momentum equations, the gas velocities will be determined from a slip velocity relation (Eq. 4.12). For small void ratios (i.e. dilute dispersed phase) the gas velocities can be calculated in the radial-direction as

$$
\begin{equation*}
v_{s}=0 \quad \text { or } \quad v_{\ell}=v_{g} \tag{4.18}
\end{equation*}
$$

and the axial direction as

$$
\begin{equation*}
u_{g}=u_{s}+u_{e} \tag{4.19}
\end{equation*}
$$

where the slip velocity $u_{s}=U_{b o}(1-a)$

Ub. is the terminal velocity of an isolated bubble in an infinite liquid medium. The effect of void ratio, $a$, on the slip velocity as given in Ea. 4.20 is suggested by Wallis(1962). Ubo can be calculated by equating the drag force to the difference of the buoyancy force and the weight of the bubble. With the drag relation Eq.4.16. this force balance results in

$$
\begin{equation*}
u_{t a}=\left[\frac{4}{3 a} \frac{\left(\rho_{\ell}-\rho_{g}\right)^{\prime}}{\rho_{\ell}} g d_{b}\left(\frac{\rho_{\ell} d_{b}}{\mu_{\ell}}\right)^{b}\right]^{1 /(z-b)} \tag{4.20}
\end{equation*}
$$

For example with $b=1$ and $a=24$, i.e., Stokes range Eg. 4.20 reduces to

$$
\begin{equation*}
U_{b=}=\frac{1}{18} \frac{\left(\rho_{\ell}-\rho_{g}\right)}{\mu_{\ell}} g d_{b}{ }^{2} \tag{4.21}
\end{equation*}
$$

If the water (or liquid) is not pure, the degree of contamination may have significant influence on $U_{b o}$ For this, the empirical curves presented by Clift et al. (1978) can be used.

An alternative is to use the terminal velocity relations presented by Hewitt (1982, chapter 2) where the terminal velocity of bubbles in clean fluids is expressed as a function of $R e_{b}$ and the Galileo number $G_{a}=g \mu_{\ell} / \rho_{\ell} \sigma^{3}$.

## Equation for void Fraction

The void ratio distribution, $a(x, r)$, is not empirically specified. So the distribution function for $\alpha$ is determined analytically in the numerical simulation. It is assumed in the present work that $\alpha(x, r)$ is
not varied in the vertical or axial direction, and thus it may be prescribed in the radial direction by either a parabolic furction distribution

$$
\begin{equation*}
\alpha(r)=a_{0}\left(1-\frac{r^{2}}{R^{2}}\right) \tag{4.22}
\end{equation*}
$$

or a cosine function distribution

$$
\begin{equation*}
\alpha(r)=\alpha_{c}(1+\operatorname{Cos}(\pi r / R j) \tag{4.23}
\end{equation*}
$$

Here $R$ is the tube radius, and $\alpha_{c}$ is the centerline value of $\alpha$ which will be related to the total gas hold-up $\alpha_{T}$ and hence to the air flow rate $Q_{a}$. Based on the experiments situation, a smoothly varying function of the form

$$
\begin{equation*}
\alpha(r)=0.5 \alpha_{C}\left(1+\operatorname{Cos}\left(\pi r / R_{S}\right)\right) \quad r \leqslant R_{S} \tag{4.24}
\end{equation*}
$$

with

$$
\alpha(r)=0 . \quad r>R_{S}
$$

was selected for the simulations. Here $R_{S}$ is the radius of the bubble street measured from the center of the column reactor. The center line value, ${ }^{\alpha} c$, was determined from conservation of mass for the gas phase, i.e.,

$$
\begin{equation*}
Q_{a}=\int_{0}^{R} 2 \pi \alpha(r) u_{q}(x, r) r d r \tag{4.25}
\end{equation*}
$$

### 4.3 Experimental Situation

A laminar liquid circulation and bubble street formation were investigated in a Quickfit glass column (Rietema and Ottengraf, 1970). The geometric configuration for the glass column is shown in Fig.4.1. The experimental conditions for the case simulated numerically are liquid density $\rho_{\ell}=1153 \mathrm{~kg} / \mathrm{m}^{3}$, liquid viscosity $\mu_{\ell}=350 \mathrm{cp}(0.35 \mathrm{~kg} / \mathrm{m}-\mathrm{s})$, air
flow rate $Q_{a}=11.4 \mathrm{~cm}^{3} / \mathrm{s}$, gas hold-up $\varepsilon_{g}=74 \mathrm{~cm}^{3}$, bubble diameter $d_{b}=0.54$ cm and bubble street diameter $D_{S}=10.0 \mathrm{~cm}$. The liquid used in the experiments was a glycerol water solution. The glass column had a diameter of 22 cm and a height of 122 cm . Initially the column was filled with the liquid solution up to a depth of 80 cm . If the gas hold-up of $74 \mathrm{~cm}^{3}$ is added to the liquid volume, the total mixture volume requires a column height of 80.195 cm . This value was used in the simuiations. Air Dubbles were formed by means of injection needles. According to experimenis, the vertical baffles were placed along the wall, thus a reasonably symmetricai street could be created. The effect of baffles is not considered at present study.

### 4.4 Computational Details

The mathematical model has been incorporated in a readily available computer code, TEACH (Gosman and Ideriah, 1976; Durst and Loy, 1984). The code is based on the finite volume approach (see for example Patankar, 1980) and it is suitable for numerical solution of incompressible, steady, single phase flow problems. All the necessary modifications have been made to include the second phase in the calculation procedure. At present the computer program can be used to calculate both components of the liquid velocity distribution in an axisymmetric configuration, as well as the pressure distribution for any given void fraction radial distribution.

The calculation domain was fixed at 80.2 cm in the axial and 11 cm in the radial directions; due to symmetry, only half of the column needs to be considered in the simulations. The uniform grid distribution is used in axial and radial directions with $\Delta x=0.04 \mathrm{~m}$ and $\Delta r=0.01 \mathrm{~m}$ for
testing of the numerical procedure. An adapted grid distribution should be used for more accurate calculation, since the computer code is able to accommodate a variable step size.

Initially, the liquid velocity field is not known. The gas velocity, $u_{\mathrm{g}}$, was calculated from
$u_{g}=u_{\ell}+u_{S}$
where $u_{\ell}$ is liquid velocity and $u_{s}$ is the slip velocity between the two phases. To start the calculations the liquid velocity was set equal to zero and the slip velocity was calculated from $u_{s}=U_{b \infty}(1-\alpha), U_{b o}$ is prescribed as a function of the bubble Reynolds number, $U_{b o}=5.23 \mathrm{~cm} / \mathrm{s}$ calculated by Eq.4.21. For the conditions summarized above, an $\alpha_{c}$ value of 0.0985 was calculated with $u_{\ell}=0$. Once $\alpha_{c}$ is determined, $\alpha(r)$ can he calculated by Eq.4.24 and the resulting $\alpha$ distribution at $x=0$ is depicted in Fig. 4.2. Then the computer program continues to solve the liquid momentum equations for $u_{\ell}$ and $v_{\ell}$, and solve liquid continuity equations for pressure. The liquid velocities are corrected aiter the pressure calculation. The result indicates that the solution converges after about 300 iteration steps. A relaxation factor of 0.7 was used for the $\alpha(r)$ calculation in the first 100 iteration steps.


#### Abstract

4.5 Results and Discussion

Rietema and Ottengraf (1970) have presented the velocity profilas from their experimental work, see Fig.4.3. The velocity profiles were measured half way up the column height (at $x=40 \mathrm{~cm}$ ) by following very small dispersed air bubbles, which move at the local liquid velocity. The operating conditions used for the numerical calculations are the same as the experiments: the dimensionless air flow rate


$\mathrm{Q}_{\mathrm{a}}=\left(8 \mu / \pi \rho_{\ell} \mathrm{g}^{+}\right) \mathrm{Q}_{\mathrm{a}}=0.3874 \times 10^{-6}$, where D is column diameter, and $Q_{a}=11.4 \mathrm{~cm}^{3} / \mathrm{s}$, air flow rate; Diameter ratio $\dot{=}=D_{s} / D=0.454$; and dimensionless slip velocity $\left(\mu_{\ell} / \rho_{\ell} g D^{2}\right) u_{S}=0.428 \times 10^{-4}$. The bubble Reynoids number Reb is about 0.9 according to the calculation, so the Stokes equation can be used for such a laminar flow. The void fraction $a(x, r)$ is calculated at each grid point of the cross section along the axial direction. Fig. 4.2 shows the result of the void fraction distributions. The curve at $x=0$ is the void fraction distribution imposed at the inlet of the column. The void fraction distribution changes gradually, but after half way up the column $(x=40 \mathrm{~cm})$, the distribution seems unlikely to change any more. The liquid velocity profile from the experiments is shown in Fig.4.3, and the liquid velocity profiles from the calculation are shown in Fig. 4.4. By comparing the results at $x=40 \mathrm{~cm}$, the magnitude of liquid velocity in the calculation at the reactor center line is about twice that in the experiment, however the shape of velocity profile and magnitude of liquid velocity near the wall are close tc that from the experiment.

The calculated stream function contours for the liquid phase are plotted in Fig.4.5. This stream function is defined as

$$
\begin{equation*}
\dot{\psi}=2-\int a \rho_{\ell} u_{\ell} r d r \tag{4.26}
\end{equation*}
$$

The result has been improved and there is no flow separation predicted. It is seen that the usual counter clockwise recirculation pattern is predicted correctly for the liquid phase.

### 4.6 Conclusions

Progress has been made in the numerical simulation for the bubbly flow in a vertically situated circular reactor.

The mathematical model for liquid-gas two-phase flow has been developed. A slip velocity relation is prescribed empirically from which the gas phase veiocity is calculated. The results show that the simplified model for the gas phase is workable for the general prediction.

The void fraction $\alpha(r)$ is described by a smoothly varying cosine function. The void fraction distribution function is a key point in the connection of the liquid and gas phases calculations. The results from the liquid velocity profile has indicated that the function for the $\alpha(r)$ distribution is in good agreement with the experiments. For future investigations, the $\alpha(r)$ function will be tested and matched more realistically to the experimental situation, such as including the baffles effect.

The numerical results show the correct patterns and shapes for the stream lines and liquid velocity profiles in comparison of the experimental results, but there is the discrepancy in the magnitude of the liquid velocity at the centerline of the column.

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## VOIDAGE PROFILE(Probe @ $1 / 2$ tt)

1 it of Water 40 cfm


Figure 2.1 Voidage Proftle at $6^{\circ \prime}$ ( 1 ft . of water and $Q_{a}=4.6 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ 1 ft)



Figure 2.2 Voldage Profile at $12^{\prime \prime}$ ( 1 ft . of water and $\mathrm{Q}_{\mathrm{a}}=4.6 \mathrm{CFM}$ )

VOIDAGE PROFILE(Probe © $1 / 2 \mathrm{ft}$ |
1 ft of Water 73 cfm


## VOIDAGE PROFILE(Probe @ 1 ft)



Figure 2.4 Voidage Profile at 12 " ( 1 ft . of water and $Q_{g}=7.3 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ $1 / 2$ ft)



Figure 2.5 Voldage Profile at $6^{\prime \prime}\left(1 \mathrm{ft}\right.$. of water and $\left.Q_{9}=10.4 \mathrm{CFM}\right)$

## VOIDAGE PROFILE(Probe @ 1 ft )



Figure 2.6 Voidage Profile at 12 " ( 1 ft . of water and $\mathrm{Q}_{\mathrm{g}}=10.4 \mathrm{CFM}$ ).

## VOIDAGE PROFILE(Probe @ $1 / 2 \mathrm{ft}$ )



Figure 2.7 Voidage Profile at $6^{\prime \prime}$ (2 ft. of water and $Q_{g}=4.6 \mathrm{CFM}$ )

VOIDAGE PROFILE(Probe @ 1 ft)


Figure 2.8 Voidage :Profile at 12 " (2 ft. of water and $Q_{g}=4.6 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ 1.5 ft )

2 ft of Water 940 cfm


Figure 2.9 Voidage Profile at $18^{\prime \prime}$ (2 ft. of water and $Q_{g}=4.6 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ 2 ft )



Figure 2.10 Voidage Proftle at 24" (2 ft. of water and $Q_{g}=4.6 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ $1 / 2 \mathrm{ft})$



Figure 2.11 Voidage Profile at $6^{\prime \prime}$ (2 ft. of water and $Q_{g}=7.3 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Prob/a @ 1 ft)

2 ft of Water 73 cf


Figure 2.12 Voidage Profile at 12 (2 ft. of water and $Q_{g}=7.3 \mathrm{cFM}$ )

## VOIDAGE PROFILE(Probe @ 1.5 ft )



Figure 2.13 Voldage Profile at 18" (2 ft. of water and $Q_{g}=7.3 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ 2 ft)



Figure 2.14 Voidage Profile at $24^{\prime \prime}$ (2 ft. of water and $Q_{g}=7.3 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ $1 / 2 \mathrm{ft})$



Figure 2.15 Voldage Profile at $6 "\left(2 \mathrm{ft}\right.$. of water and $Q_{\mathrm{g}}=10.4 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ 1 ft)



Figure 2.16 Voidage Proflle at 12 " (2 ft. of water and $Q_{g}=10.4 \mathrm{CFM}$ )

VOIDAGE PROFILE(Probe @ 1.5 ft )


Figure 2.17 Voidage Proflle at $18^{\prime \prime}$ ( 2 ft . of water and $Q_{g}=10.4 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ 2 ft)



Figure 2.18 Voldage Profile at $24^{\prime \prime}$ ( 2 ft . of water and $Q_{g}=10.4 \mathrm{cFM}$ )

VOIDAGE PROFILE(Probe @ 1.5 ft )


Figure 2.19 Voldage Profile at $18^{\prime \prime}$ ( 3 ft . of water and $Q_{9}=4.6 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ 2 ft)



Figure 2. 20 Voldage Profile at $24^{\prime \prime}$ (3 ft. of water and $Q_{g}=4.6 \mathrm{GFM}$ )

## VOIDAGE PROFILE(Probe @ 3 ft)



Figure 2.21 Voidage Profile at $36^{\prime \prime}$ ( 3 ft . of water and $Q_{g}=4.6 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ 1.5 ft )



Figure 2.22 Voidage Profile at $18^{\prime \prime}$ (3 ft. of water and $Q_{g}=7.3 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ 2 ft)



Figure 2.23 Voldage Profile at $24^{\prime \prime}$ (3 ft. of water and $Q_{g}=7.3 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ 3 ft )



Figure 2.24 Voidage Profile at 36 " (3 ft. of water and $Q_{g}=7.3 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ 1.5 ft )



Figure 2.25 Voidage Profile at $18^{\prime \prime}$ (3 ft. of water and $Q_{y}=10.4 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ 2 ft$)$



Figure 2.26 Votdage Proflle at $24^{\prime \prime}$ ( 3 ft . of water and $\mathrm{Q}_{\mathrm{g}}=10.4 \mathrm{CFM}$ )

## VOIDAGE PROFILE(Probe @ 3 ft)



Figure 2.27 Voidage Profile at $36^{\prime \prime}$ (3 ft. of water and $Q_{g}=10.4 \mathrm{CFM}$ )


Figure 2.28 Resistance probes for cross-correlation

## CROSS-CORRELATION



Figure 2.29 Experimental trace from cross-correlation probes

## SHEAR STRESS PROFILE(1 ft of water)



Figure 2.30 Shear stress proflle at $6^{\prime \prime}$ ( 1 ft . of water and $Q_{g}=4.6 \mathrm{CFM}$ )

VOIDAGE PROFILE(1 ft of water)


Figure 2.31 Voidage profile at $6^{\prime \prime}\left(1 \mathrm{ft}\right.$. of water and $\left.Q_{g}=4.6 \mathrm{CFM}\right)$

## DENSITY PROFILE(1 ft of water)




Figure 2.32 Mixture density proflle at $6^{\prime \prime}\left(1 \mathrm{ft}\right.$. of water and $\left.Q_{g}=4.6 \mathrm{CFM}\right)$

## LIQUID VELOCITY PROFILE(1 FT OF WATER)

PROFILE AT $\sigma^{\circ}\left(T W=-13018 N / M{ }^{2} 2\right)$


Figure 2.33 Liquid velocity profile at $6^{\prime \prime}$ ( 1 ft . of water and $\mathbf{a}_{g}=4.6 \mathrm{CFM}$ )

## LIQUID VELOCITY PROFILE(1 FT OF WATER)



Figure 2.34 Liquid velocity profile at $6^{\prime \prime}$ ( 1 ft . of water at 3 flowrates)


Figure 3.1 Schematic of hexagonal cross-section bubble column for laser doppler velocimeter measurements.


Figure 3.2 Preliminary measured mean vertical velocity profiles in hexagonal bubble column versus depth at three different radii; air flow rate $=2 \mathrm{SCFH}$.


Figure 3.3 Preliminary measured RMS vertical velocity profiles in hexagenal bubble column versus depth at three different radii: air flow rate $=2$ SCFH.


Fig. 4.1 Geometry of the bubble column


Fig. 4.2 Void fraction distribution as a function of the radial distance


$$
\begin{aligned}
& q_{d}=0.3834 \times 10^{-6} \\
& \phi=0.4108 \times 10^{-4}
\end{aligned}
$$

$$
\varepsilon=0.0118
$$

$$
\delta=0.46
$$

Fig. 4.3 Experimentally determined velocity profile, from Rietema and Ottengraf (1970).


Fig. 4.4 Calculated Liquid Velocity Profiles


## APPENDIX A Cross－Correiation Program

```
    DEFINT :-
        O:H FPGAY(13000). IA( 302), IV(11)
    FOf: KK= = io ic
## ETETAT = 0
    CPLL IN:T:FGIこEIEPSTAT,
        :E ERSTA: = 0 MiEN 1ON
            GGINT THE ENPNF :S : EPSTGIT
            infut as
            CuT0
:00 : %Mロ4 = 1
        SOAFO= =1
        ミRミ㣙 = 0
        MHAT = 3
        MulLT = 50
        TowGE = 10
        CTUNTT = :E,000
        ra:N = i
        IALL SCAN:SISILOMAN. EOARD. POHAN. RANGE. MLLT, COUNT. GAIN. SRSTAT)
        IF EPSTAT = O THEN 200
            DFINT THE ETROR IS*: SRSTAT
            :HPuT CS
            OTO 100
2OC IM:PUT CCLLECTION OF DATA BEGINS ON ANY INPUT": ES
        PRENT TIMER
        LOMAN = :
        ENSTAT = 0
            ES'L SCNH(LEHAN. ARPAY:1), ERSTAT;
        IF SRETAT = O THEN 300
            ORINT THE ERPOR 15*: ERSTAT
            EnPuT CS
            00TO <00
3NE L.CHAN = :
            EFSTAT = 0
S:C CALL MEOK(LCHAN. gRSTAT,
            IF ERSTAT = 117 THEN 000
            =拃 310
300 FOINT CCLLECTION OF DATA COPPLETE-
    PRINT TIMEF
    PEAD AS
    TNEN "O",*1.AS
     TOP: = 1 TO ONNT
            PRINT #1, APRAY(I)
    NEKT I
    ClOこE #1
        TOP K = : TO :52
            L=K-5!
            ZA(N: = O
            IFK = S: THEN EOTO 500
            M = SONT - 201
            N = 2
            FOP : = N TOM STEP 2
```



```
            NEOT :
            00TO 500
```

    \(N=1+200\)
        9 : GORJT • 1
    

MET:
AEXT
$\because=E$
Tars $=0$

:F zal:
TEMF = ZP: ( )
ジいい」 =
550 NEMT:
:

03 ro $=0$

FEC NEMT AK
data randata"
Esic

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[^0]:    In addition, hardware and software has been developed to infer vertical bubble velocities in the column. This is achieved by cross-correlating the two voltage signals from two separate resistance probe tips, the one tip being located directly above the other. A probe with vertical tip spacing of $3 / 8$ incin (as shown in Figure 2.28 ) has been manufactured, and the two tips have been tested separately in air and water. Data is sampled rapidly (at 2000 Hz ) from the probes, and stored in computer memory. The two signals are then cross-corre!ated to find the best-fit time lag between bubbles reaching the lower probe and the same bubbles reaching the upper probe. A copy of the cross-correlation program is attached to this report in Appendix A. Figure 2.29 shows high resolution traces of voltage from the upper and lower probes on the same set of axes. The troughs in each trace indicate bubble presence, and the time delay between traces is evident. The accuracy of the cross-correlation scheme and the probe hardware has been determined by placing the probes in a stream of bubbles rising from a point source below the probes. One would expect the measured velocity of this string of bubbles to be slightly greater than the rise velocity of a lone bubble in water, which is known to be about $240 \mathrm{~mm} / \mathrm{sec}$. for bubbles or size between the Stokes and bubble cap regimes. The program showed a satisfactory rise velocity of $255 \mathrm{~mm} / \mathrm{sec}$. It can now be used with confidence to find the bubble velocity in circulating systems. These bubble velocities can then be compared with the liquid veiocities predicted by the numerical modeling, and by the one-dimensional models discussed previously in this research. The one-dimensional model has recently been checked

