Appendix A

Numerical Solution for Axial Solids Concentration Profile

Based on a one-dimensional axial solids dispersion model that assumes steady-state conditions, our model is essentially a mass balance of the solid particles at any cross section. The equation is as follows:

$$\frac{u_\ell}{1 - \varepsilon_g} (C - C_f) - E_{zp} \frac{dC}{dL} - V_p C = 0 \qquad (1)$$

where u_ℓ = superficial liquid velocity (ft/sec); ε_g = average gas holdup; C = axial solids concentration in slurry (lbm/ft^3); C_f feed slurry concentration (lbm/ft^3); E_{zp} = axial solids dispersion coefficient (ft^2/sec); L = axial height (ft); V_p = particle settling velocity (ft/sec).

Rearranging equation (1) by dividing by E_{zp} gives;

$$\frac{u_\rho}{E_{zp}(1 - \varepsilon_g)} (C - C_f) - \frac{dC}{dL} - \frac{V_p}{E_{zp}} C = 0 \qquad (2)$$

The values of u_ℓ, E_{zp}, C_f, and V_p/E_{zp} are required for this model.

From the report (Tables 6-14) on solids dispersion, we obtained values of V_p/EH_{zp} for different gas velocities and different average solids concentrations. Usually, V_p/E_{zp} decreased as the average concentration of solids in the column increased. This was attributed to a decrease in V_p as the concentration increased.

No direct means were used to measure E_{zp}, but we assumed that:

$$E_{zp} = E_{zl} \text{ (liquid dispersion coefficient)} \qquad (3)$$

The basis for this assumption is that the particles still follow the movements of the liquid fluctuations to a reasonable degree. Also, another assumption that was made was that $V_p/E_{zp} = V_p/E_{zp}$ (c), i.e.,

function of local concentration, c. Hence this ratio will vary with column height. The value of E_{z1} was already obtained in experiments described in the report in the liquid dispersion section (Tables 3-5).

Due to the possibility of random errors in the measurement of the feed concentration (C_F) the C_F value used in this model was optimized to provide the best agreement between the integrated experimental data of sand in the column and the calculated amount of sand by the model. We used an iterative method that was usually fast; on the average, about 13 iterations were required to attain a relative accuracy of 10^{-4}; i.e.,

$$\frac{W_{Calcd} - W_{act.}}{W_{act}} \leq 10^{-4} \qquad (4)$$

where $W_{calcd.}$ and $W_{act.}$ are amounts of solids retained in the column by calculation and actual measurement, respectively.